MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 9

Duality of LP and its interpretation continued

Example:

Obtain the dual of the following primal problem

Minimize $Z = 3x_1 - 2x_2 + x_3$

Subject to the constraints

$2x_1 - 3x_2 + x_3 \leq 5 \rightarrow (1)$

$4x_1 - 2x_2 \geq 9 \rightarrow (2)$

$-8x_1 + 4x_2 + 3x_3 = 8 \rightarrow (3)$

$x_1 \geq 0, x_2 \geq 0, x_3$ is unrestricted

As x_3 is unrestricted let $x_3 = x_{31} - x_{32}$ where $x_{31} \geq 0, x_{32} \geq 0$

On substituting x_{31} - x_{32} for x_3 and considering equality constraints the given problem becomes

Minimize $Z = 3x_1 - 2x_2 + x_{31} - x_{32}$

Subject to the constraints

$2x_1 - 3x_2 + x_{31} - x_{32} \leq 5 \rightarrow (1)$

$4x_1 - 2x_2 \geq 9 \rightarrow (2)$

$-8x_1 + 4x_2 + 3x_{31} - 3x_{32} \geq 8 \rightarrow (3)$

$-8x_1 + 4x_2 + 3x_{31} - 3x_{32} \leq 8 \rightarrow (4)$

$x_1 \geq 0, x_2 \geq 0, x_{31} \geq 0, x_{32} \geq 0$

Multiplying constraints 1 and 4 by -1 to convert ≤ to ≥ we get

Minimize $Z = 3x_1 - 2x_2 + x_{31} - x_{32}$

Subject to the constraints

$-2x_1 + 3x_2 - x_{31} + x_{32} \geq -5 \rightarrow (1)$

$4x_1 - 2x_2 \geq 9 \rightarrow (2)$

$-8x_1 + 4x_2 + 3x_{31} - 3x_{32} \geq 8 \rightarrow (3)$

$8x_1 - 4x_2 - 3x_{31} + 3x_{32} \geq -8 \rightarrow (4)$

$x_1 \geq 0, x_2 \geq 0, x_{31} \geq 0, x_{32} \geq 0$

Hence the dual of the problem is

Maximize $W = -5y_1 + 9y_2 + 8y_3 - 8y_4$

Subject to the constraints

$-2y_1 + 4y_2 - 8y_3 + 8y_4 + \leq 3 \rightarrow (1)$

$3y_1 - 2y_2 + 4y_3 - 4y_4 \leq -2 \rightarrow (2)$

$-y_1 + 3y_3 - 3y_4 \leq 1 \rightarrow (3)$

$y_1 - 3y_3 + 3y_4 \leq -1 \rightarrow (4)$

$y_1 \geq 0, \ y_2 \geq 0, \ y_3 \geq 0, \ y_4 \geq 0, \ y_5 \geq 0 \rightarrow$ non negativity restrictions

where y_1, y_2, y_3, y_4 are dual variables associated with primal constraints 1,2,3 and 4 respectively

Putting $y_3 - y_4 = y$ we get

Maximize $W = -5y_1 + 9y_2 + 8y$

Subject to the constraints

$-2y_1 + 4y_2 - 8y \leq 3 \rightarrow (1)$

$3y_1 - 2y_2 + 4y \leq -2 \rightarrow (2)$

$-y_1 + 3y \leq 1 \rightarrow (3) \qquad \longrightarrow -y_1 + 3y = 1$

$y_1 - 3y \leq -1 \rightarrow (4)$

$y_1 \geq 0, y_2 \geq 0, y$ is unrestricted

where y_1, y_2, y are dual variables associated with primal constraints 1,2 and 3 respectively

- TILL WE MEET AGAIN IN THE NEXT CLASS.........