MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 8

Duality of LP and its interpretation

DUALITY IN LINEAR PROGRAMMING

- For every LP problem(primal), there exists a related unique LP problem involving the same data which also describes the original problem. This is called the ***dual*** problem.
- If the primal contains n variables and m constraints, the dual will contain m variables and n constraints.
- The maximization problem in the primal becomes the minimization problem in the dual and vice versa.
- The maximization problem has ≤ constraints while the minimization problem has ≥ constraints.
- The coefficients in the objective function of the primal become the RHS constants in the constraints of the dual.
- The constants on the RHS of the constraints of the primal become the coefficients of the objective function in the dual.

DUALITY THEOREMS

- ❖The dual of the dual is the primal.
- ❖The value of the objective function Z for any feasible solution of the primal is ≤ the value of the objective function W for any feasible solution of the dual.
- ❖If either the primal or the dual problem has an unbounded solution, then the solution to the other problem is infeasible
- ❖If both the primal and the dual problems have feasible solutions then both have optimal solutions and max Z=min W
- ❖Complementary slackness theorem:
  - a) If a primal variable is positive then the corresponding dual constraintis an equation at the optimum
  - b) If the primal constraint is a strict inequality, then the corresponding dual variable is zero at the optimum
  - c) If a dual variable is positive then the corresponding primal constraint is an equation at the optimum
  - d) If a dual constraint is a strict inequality then the corresponding primal variable is zero at the optimum.

LPP-DUALITY

Example:

Construct the dual of the problem

Minimize $Z=3x_1 -2x_2 +4x_3$

Subject to the constraints

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ → non negativity restrictions

$3x_1+5x_2+ 4x_3 \geq 7$ →(1)

$6x_1+x_2+ 3x_3 \geq 4$ →(2)

$7x_1-2x_2 - x_3 \leq 10$ →(3)

$x_1-2x_2+ 5x_3 \geq 3$ →(4)

$4x_1+7x_2 - 2x_3 \geq 2$ →(5)

In a minimization problem, all constraints should be of the ≥ type

Therefore multiplying constraint 3 by – we get

$-7x_1+2x_2 + x_3 \geq -10$

LPP-DUALITY

Hence the dual of the problem is

Maximize $W=7y_1 + 4y_2 -10y_3 +3y_4 +2y_5$

Subject to the constraints

$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0,\ y_5 \geq 0$ → non negativity restrictions

$3y_1 + 6y_2 -7y_3 +y_4 +4y_5 \leq 3$ →(1)

$5y_1 + y_2 +2y_3 -2y_4 +7y_5 \leq -2$ →(2)

$4y_1 + 3y_2 +y_3 +5y_4 -2y_5 \leq 4$ →(3)

where y_1, y_2, y_3, y_4, y_5 are dual variables associated with primal constraints 1,2,3,4,and 5 respectively

LPP-Equality Constraints in Duality

Example:

Obtain the dual of the following primal LP problem

Minimize $Z = x_1 + 2x_2$

Subject to the constraints

$x_1 \geq 0, x_2 \geq 0$ → non negativity restrictions

$2x_1 + 4x_2 \leq 160$ →(1)

$x_1 - x_2 = 30$ →(2)

$x_1 \geq 10$ →(3)

LPP-Equality Constraints in Duality

Because the problem is a minimization problem, changing all ≤ type constraints to ≥ type and = type constraints to two constraints of ≤ and ≥ type we get

Minimize $Z=x_1 +2x_2$

Subject to the constraints

$x_1 \geq 0, x_2 \geq 0$ → non negativity restrictions

$-2x_1-4x_2 \geq -160$ →(1)

$x_1-x_2 \geq 30$ →(2)

$x_1-x_2 \leq 30$ →(3)

$x_1 \geq 10$ →(3)

LPP-EQUALITY CONSTRAINTS IN DUALITY

Hence the dual of the problem is

Maximize $W=-160y_1 + 30y_2 - 30y_3 + 10y_4$

Subject to the constraints

$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$ → non negativity restrictions

$-2y_1 + y_2 - y_3 + y_4 \leq 1$ →(1)

$-4y_1 - y_2 + y_3 \leq 2$ →(2)

Taking $y_2 - y_3 = y'$ the LP problem becomes

Maximize $W=-160y_1 + 30y' + 10y_4$

Subject to the constraints

$-2y_1 + y' + y_4 \leq 1$ →(1)

$-4y_1 - y' \leq 2$ →(2)

$y_1 \geq 0$, y_4 unrestricted

- TILL WE MEET AGAIN IN THE NEXT CLASS……….