MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 7

Linear Programming: Big M Method continued

LPP-BIG M METHOD APPLICATION

Example:

Minimize $Z=2x_1 + 3x_2$

Subject to

$x_1 \geq 0, x_2 \geq 0$ → non negativity restrictions

$x_1 + x_2 \geq 5$ →(1)

$x_1 + 2x_2 \geq 6$ → (2)

Converting the maximization problem to a minimization one by multiplying by -1

Maximize $Z' = -2x_1 - 3x_2$

LPP-BIG M METHOD APPLICATION

- **Introducing slack/surplus variables to convert inequalities into equalities**

Maximize $Z' = -2x_1 - 3x_2 + 0s_1 + 0s_2$

Subject to

$x_1 + x_2 - s_1 = 5 \rightarrow (1)$

$x_1 + 2x_2 - s_2 = 6 \rightarrow (2)$

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0 \rightarrow$ non negativity restrictions

Putting $x_1=0$ and $x_2=0$ in constraints 1 and 2 we get

$s_1 = -5, s_2 = -6$

LPP-BIG M METHOD APPLICATION

- Since negative values for the slack variables are not allowed we introduce artificial variables A_1 and A_2.

$x_1+x_2- s_1+A_1 = 5 \rightarrow (1)$

$x_1+2x_2- s_2 +A_2 = 6 \rightarrow (2)$

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0 , s_2 \geq 0, A_1 \geq 0, A_2 \geq 0 \rightarrow$ non negativity restrictions

Putting $x_1=0$, $x_2=0$, $s_1=0$ and $s_2=0$ in constraints 1 and 2 we get

$A_1 = 5, A_2 = 6$

To eliminate artificial variables from the final solution (because artificial variables with values greater than zero destroy the equality required by the LP Model), a large *penalty* or negative value(-M) is assigned to the artificial variables in the objective function

Maximize $Z' = -2x_1 - 3x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

LPP-BIG M METHOD APPLICATION

Objective function →	c_j	-2	-3	0	0	-M	-M		
e_i	csv	x_1	x_2	s_1	s_2	A_1	A_2	b_i	ϴ
-M	A_1	1	1	-1	0	1	0	5	5
-M	A_2	1	2	0	-1	0	1	6	3
$Z_j=e_ia_{ij}$		-2M	-3M	M	M	-M	-M		
C_j- Z_j		-2+2M	-3+3M	-M	-M	0	0		

KEY ROW

outgoingvariable A_2

KEY COLUMN

KEY ELEMENT

incoming variable X_2

LPP-BIG M METHOD APPLICATION

Objective function →	c_j	-2	-3	0	0	-M	-M		
e_i	**csv**	x_1	x_2	s_1	s_2	A_1	A_2	b_i	θ
-M	A_1	1	1	-1	0	1	0	5	
-3	x_2	1/2	1	0	-1/2	0	1/2	3	
$Z_j=e_ia_{ij}$									
$C_j- Z_j$									

Dividing the key row by 2 to get unity for key element

LPP-BIG M METHOD APPLICATION

Objective function →	c_j	-2	-3	0	0	-M		
e_i	**csv**	x_1	x_2	s_1	s_2	A_1	b_i	θ
-M	A_1	1/2	0	-1	1/2	1	2	4
-3	x_2	1/2	1	0	-1/2	0	3	6
$Z_j=e_ia_{ij}$		-M/2-3/2	-3	M	-M/2+3/2	-M		
C_j- Z_j		-2-(M+3)/2 =(-1+M)/2	0	-M	(-3+M)/2	0		

Incoming variable **x_1** **Key Element**

Outgoing variable A_1

LPP-BIG M METHOD APPLICATION

Objective function →	c_j	-2	-3	0	0		
e_i	**csv**	x_1	x_2	s_1	s_2	b_i	θ
-2	x_1	1	0	-2	1	4	
-3	x_2	1/2	1	0	-1/2	3	
$Z_j=e_i a_{ij}$		-M/2-3/2	-3	M	-M/2+3/2		
$C_j- Z_j$		-2-(M+3)/2 =(-1+M)/2	0	-M	(-3+M)/2		

Multiplying row 1 by 2 to make the key element unity

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	-2	-3	0	0		
e_i	CSV	x_1	x_2	s_1	s_2	b_i	θ
-2	x_1	1	0	-2	1	4	
-3	x_2	0	1	1	-1	1	
$Z_j=e_ia_{ij}$		-2	-3	1	1		
$C_j- Z_j$		0	0	-1	-1		

Dividing row 2 by 2 and subtracting the corresponding results from row 1

All C_j- Z_js are negative or zero. Hence optimality is reached .

Therefore x_1 =4, x_2=1 and Z= 2 x_1+ 3 x_2= 2X4+3X1=11

- Till we meet again in the next class..........