MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 17

Game Theory

Two Person Zero Sum Game

- Only two players are involved.
- Each player has a finite number of strategies to use.
- Each specific strategy results in a payoff.
- Total payoff to the two players at the end of each play is zero i.e. gain of one player equals the loss of the other

TWO PERSON ZERO SUM GAME

- **Player** :- A competitor in a game.
- **Strategy** :- Set of alternative courses of actions available to a player.
- **Pure Strategy** :- A strategy a player selects each time with complete knowledge of opponent's action with an objective to maximize gain or minimize loss.
- **Optimum Strategy** :- Strategy that puts the player in a preferred position irrespective of the competitor's strategy.
- **Mixed Strategy** :- When a combination of strategies is used by a player and he has little knowledge about the opponent's strategy, a probabilistic situation with the objective of maximizing expected gains or minimizing expected losses, arises.
- **Value of the game** :- Expected payoff when all players follow the optimum strategy.
- **Fair Game** :- When the value of the game is zero.
- **Unfair Game** :- When the value of the game is not zero.

MAXIMIN-MINIMAX PRINCIPLE

Example:

		Player B		**Row Min**
		B_1	B_2	
Player A	A_1	9	2	2
	A_2	8	*6	6(Max)
	A_3	6	4	4
Column Max		9	6(Min)	

SADDLE POINT

- **A** follows maximin strategy while **B** follows minimax strategy
- A plays strategy $\mathbf{A_2}$ while B plays strategy $\mathbf{B_2}$
- In a fair game, maximin value=minimax value=0
- A game is said to have a **saddle point** if the maximin value=minimax value

MAXIMIN-MINIMAX PRINCIPLE

	Player B			Row Min
Player A	2	4	5	2
	10	7	q	7
	4	p	6	?
Column Max	10	7	?	

SADDLE POINT

Example :Find the range of values of p and q which render the entry (2,2) a saddle point for the game.

Solution:

If entry (2,2) i.e. 7 has to be the saddle point then it must be the column maxima and p≤ 7.

Similarly if entry (2,2) i.e. 7 has to be the saddle point then it must be the row minima and q≥ 7.

Reducing a Game by Dominance & Mixed Strategies

- ***Dominance rule for column***: Every value in the dominating column(s) must be less than or equal to the corresponding value of the dominated column.
- ***Dominance rule for row***: Every value in the dominating row(s) must be greater than or equal to the corresponding value of the dominated row.
- ***Mixed Strategy***: When there is no saddle point

i. Subtract the two digits in column 1 and write the difference under column 2 ignoring sign.

ii. Subtract the two digits under column 2 and write the difference under column 1 ignoring sign

iii. Proceed similarly with the two rows.

These values are called ***ODDMENTS***

Reducing Game by Dominance & Mixed Strategies

❖Example :Two players P and Q play a game . Each of them has to choose one of the three colours white (W), black(B) and red(R) independently of the other. If both P and Q choose W,W neither wins anything. If player P selects white and Q black (W,B), P loses Rs. 2 and Q wins the same.

❖**There is no saddle point.**

❖Here column R is dominated by column B because all elements in column B are less than corresponding elements of column R.

❖Therefore column R is eliminated.

		Player Q			Row Min
		W	B	R	
Player P	W	0	-2	7	-2
	B	2	5	6	2
	R	3	-3	8	-3
Column Max		3	5	8	

REDUCING GAME BY DOMINANCE & MIXED STRATEGIES

❖Here column R is dominated by column B because all elements in column B are less than corresponding elements of column R.
❖Therefore column R is eliminated.

		Player Q	
		W	B
Player P	W	0	-2
	B	2	5
	R	3	-3

❖Now row W is dominated by row B because all elements in row W are less than the corresponding elements of row B.
❖Therefore row W is deleted.

		Player Q	
		W	B
Player P	B	2	5
	R	3	-3

Reducing Game by Dominance & Mixed Strategies

- In column W difference between two elements 2 and 3 is 1 which is written under column B.
- In column B the difference between two elements 5 and -3 is 8 which is written under column W.
- Similarly in row B difference between the two elements 2 and 5 is 3 which is written beside row R
- In row R difference between the two elements 3 and -3 is 6 which is written beside row B.

		Player Q		
		W	B	
Player P	B	2	5	6
	R	3	-3	3
		8	1	

REDUCING GAME BY DOMINANCE & MIXED STRATEGIES

Solution:

- Q should play White 8/(8+1) fraction of the time and Black 1/(8+1) fraction of the time i.e. 8/9 and 1/9 respectively.
- P should play Black 6/(6+3) fraction of the time and Red 3/(6+3) fraction of the time i.e. 6/9 and 3/9 respectively.

		Player Q		
		W	B	
Player P	B	2	5	6
	R	3	-3	3
		8	1	

Graphical Method

Example: Two firms A and B make colour and black and white television sets. Firm A can make either 150 colour sets in a week or an equal number of black and white sets making a profit of Rs. 400 per colour set and Rs. 300 per black and white set. Firm B can either make 300 colour sets or 150 colour and 150 black and white sets or 300 black and white sets per week. It also has the same profit margins on the two sets as A. Each week there is a market of 150 colour sets and 300 black and white sets and the manufacturers would share market in the proportion in which they manufacture a particular type of set. Write the payoff matrix of A per week. Obtain graphically A and B’s optimum strategies and the value of the game.

Graphical Method

Solution:

Firm A's strategies

A_1 : Make 150 colour sets

A_2 : Make 150 black and white sets

Firm B's strategies

B_1 : Make 300 colour sets

B_2 : Make 150 colour and 150 black and white sets

B_3 : Make 300 black and white sets

GRAPHICAL METHOD

Solution:

Payoff to A for strategy A_1 B_1 i.e. A plays A_1 and B plays B_1

(A's manufacture of colour sets/total manufacture) X total market of colour sets X profit per set

={150/(150+300)}X150X400=Rs. 20,000 Market share of A

Payoff to A for strategy A_1 B_2

={150/(150+150)}X150X400=Rs 30,000

Payoff to A for strategy A_1 B_3

=150X 400=Rs 60,000 because only A manufactures 150 colour sets and market demand is 150. Hence all sell

Graphical Method

Solution:

Payoff to A for strategy A_2 B_1

150X300=Rs. 45,000 Only A makes black and white sets(150). Demand is 300. Hence all 150 sets sell.

Payoff to A for strategy A_2 B_2

={150/(150+150)}X300X300=Rs 45,000

Payoff to A for strategy A_2 B_3

={50/(150+300)}X 300X300=Rs 30,000

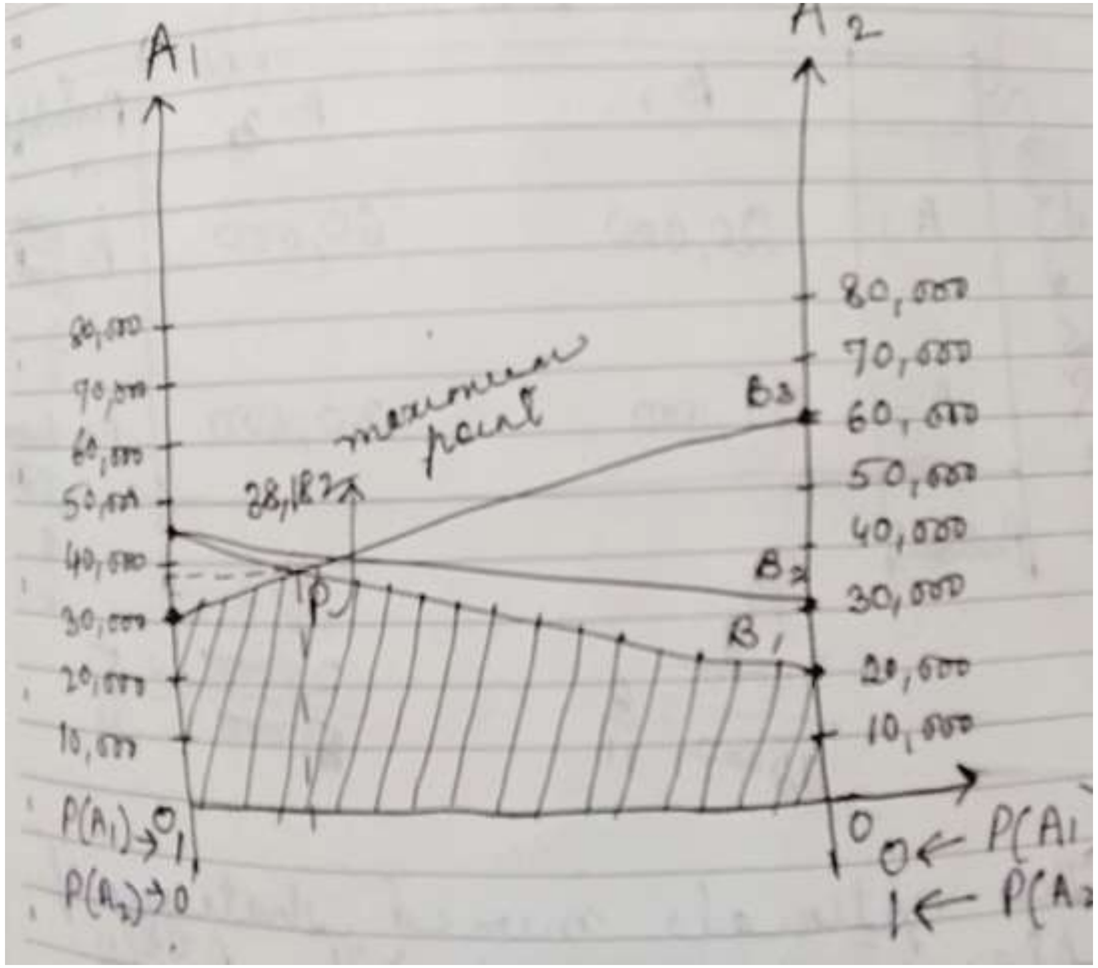
GRAPHICAL METHOD

- Since there exists no saddle point, to determine optimal mixed strategy, we plot the data on a graph.

		B's strategy			Row Min
		B_1	B_2	B_3	
A's strategy	A_1	20,000	30,000	60,000	**20,000**
	A_2	45,000	45,000	30,000	**30,000**
Column Max		**45,000**	**45,000**	**60,000**	

Graphical Method

- Since there exists no saddle point, to determine optimal mixed strategy, we plot the data on a graph.
- Lines joining payoffs on axis A_1 with payoffs on axis A_2 represent each of B's strategies.
- P represents the maximum expected payoff or value of the game for A.
- The two strategies of B, B_1 and B_3 which pass through P need to be adopted by B eliminating B_2
- The solution reduces to a 2X2 payoff matrix.



Graphical Method

- Therefore the optimal mixed strategy of player A are $A_1 \rightarrow 3/11$, $A_2 \rightarrow 8/11$ and the optimal mixed strategies of player B are $B_1 \rightarrow 6/11$, $B_3 \rightarrow 5/11$
- The value of the game from the graph is Rs. 38,182 for A
- Algebraically value of the game for A is 20,000X3/11+45,000X8/11=38,181.8181 = Rs. 38,182/-

		B's strategy		Probability
		B_1	B_3	
A's strategy	A_1	20,000	60,000	p_1=15000/50000 = 3/11
	A_2	45,000	30,000	p_2=40,000/55000 = 8/11
Probability		q_1=30000/ 55000= 6/11	q_2=25000/ 550000= 5/11	

- TILL WE MEET AGAIN IN THE NEXT CLASS……….