MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 16

Unbalanced Assignment Problems

ASSIGNMENT PROBLEMS

Example:

A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1,2,3,4,5 and 6. The distance between the cities in kilometers is shown in the matrix below. Find the assignment of trucks from cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum.

	SUPPLIERS						
Cities in Surplus		1	2	3	4	5	6
	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

ASSIGNMENT PROBLEMS

Solution:

Introducing a dummy city with surplus vehicle

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10
	Dummy	0	0	0	0	0	0

ASSIGNMENT PROBLEMS

Subtracting the lowest value of each row from all the elements of the row and allocating the zeros we get

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

Allocation is not optimal because number of rows = number of columns = 6 but number of allocations is 5

ASSIGNMENT PROBLEMS

Marking rows with no assignments and columns with zeros in marked rows

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

ASSIGNMENT PROBLEMS

Marking rows with assigned zeros in marked columns

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

ASSIGNMENT PROBLEMS

Drawing lines through marked columns and unmarked rows

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

ASSIGNMENT PROBLEMS

The smallest element in the unmarked cells is 2. Subtracting this element from cells untouched by lines and adding it to the cells at the cross section of two lines we get

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

ASSIGNMENT PROBLEMS

Subtracting the smallest element in each row from all the elements in the row we get

	Cities in Deficit						
Cities in Surplus		1	2	3	4	5	6
	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

Assignment is not optimal because rows=columns=6 and number of assignments=5

ASSIGNMENT PROBLEMS

Marking rows with no assignments and columns with zeros in marked rows we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

ASSIGNMENT PROBLEMS

Marking rows with assigned zeros in marked columns

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

ASSIGNMENT PROBLEMS

Drawing lines through marked columns and unmarked rows we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

ASSIGNMENT PROBLEMS

The smallest element in the unmarked cells is 2. Subtracting this element from cells untouched by lines and adding it to the cells at the cross section of two lines we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	8	2	7	14	10	2
	B	0	4	11	1	2	0
	C	10	7	0	5	2	6
	D	0	4	0	3	3	4
	E	5	5	4	0	6	5
	Dummy	4	0	0	0	0	2

ASSIGNMENT PROBLEMS

Subtracting smallest element in each row from all row elements and making allocations we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	0	5	12	8	0
	B	0	4	11	1	2	0
	C	10	7	0	5	2	6
	D	0	4	0	3	3	4
	E	5	5	4	0	6	5
	Dummy	4	0	0	0	0	2

Therefore no truck is supplied to city 5. Truck goes from city A to 2, B to 6, C to 3, D to 1, E to 4 and F to 5.
Minimum distance travelled is 10+12+3+6+7=38 kms

- TILL WE MEET AGAIN IN THE NEXT CLASS.....