

MB106

QUANTITATIVE TECHNIQUES

A horizontal banner with a light green background and dark green gear patterns. The text "OPERATIONS RESEARCH" is written in a bold, dark green, sans-serif font across the center.

**OPERATIONS
RESEARCH**

MODULE I

LECTURE 16

Unbalanced Assignment Problems

ASSIGNMENT PROBLEMS

Example:

A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1,2,3,4,5 and 6. The distance between the cities in kilometers is shown in the matrix below. Find the assignment of trucks from cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum.

		SUPPLIERS					
		1	2	3	4	5	6
Cities in Surplus	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

ASSIGNMENT PROBLEMS

Solution:

Introducing a dummy city with surplus vehicle

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10
	Dummy	0	0	0	0	0	0

ASSIGNMENT PROBLEMS

Subtracting the lowest value of each row from all the elements of the row and allocating the zeros we get

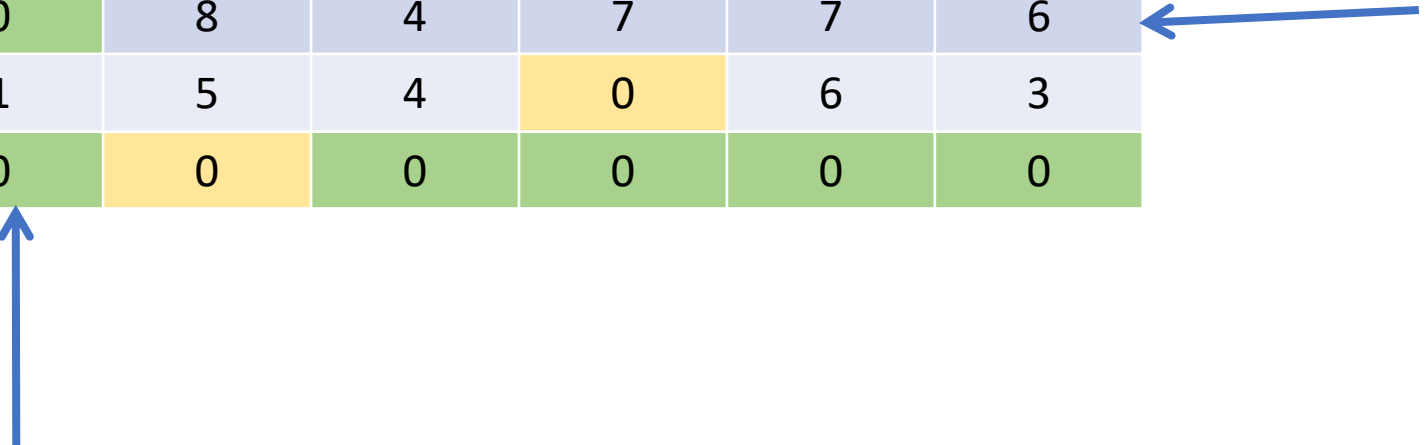
		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

Allocation is not optimal because number of rows = number of columns = 6 but number of allocations is 5

ASSIGNMENT PROBLEMS

Marking rows with no assignments and columns with zeros in marked rows

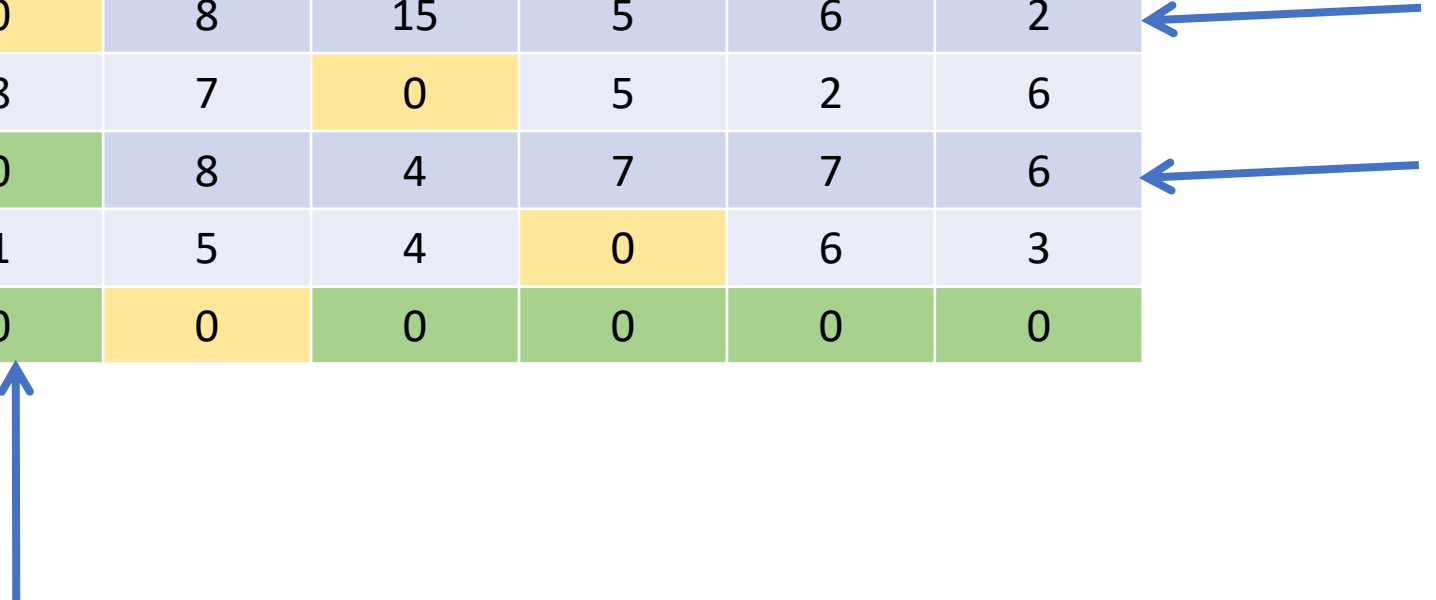
		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0



ASSIGNMENT PROBLEMS

Marking rows with assigned zeros in marked columns

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0



ASSIGNMENT PROBLEMS

Drawing lines through marked columns and unmarked rows

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	4	2	7	14	10	0
	B	0	8	15	5	6	2
	C	8	7	0	5	2	6
	D	0	8	4	7	7	6
	E	1	5	4	0	6	3
	Dummy	0	0	0	0	0	0

The diagram illustrates a transportation problem matrix. The columns are labeled 'Cities in Deficit' and numbered 1 through 6. The rows are labeled 'Cities in Surplus' and lettered A through E, plus a 'Dummy' row. Blue arrows point to the following cells: column 1, row B, and row D.

ASSIGNMENT PROBLEMS

The smallest element in the unmarked cells is 2. Subtracting this element from cells untouched by lines and adding it to the cells at the cross section of two lines we get

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

ASSIGNMENT PROBLEMS

Subtracting the smallest element in each row from all the elements in the row we get

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

Assignment is not optimal because rows=columns=6 and number of assignments=5

ASSIGNMENT PROBLEMS

Marking rows with no assignments and columns with zeros in marked rows we get

		Cities in Deficit					
Cities in Surplus		1	2	3	4	5	6
	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0



ASSIGNMENT PROBLEMS

Marking rows with assigned zeros in marked columns

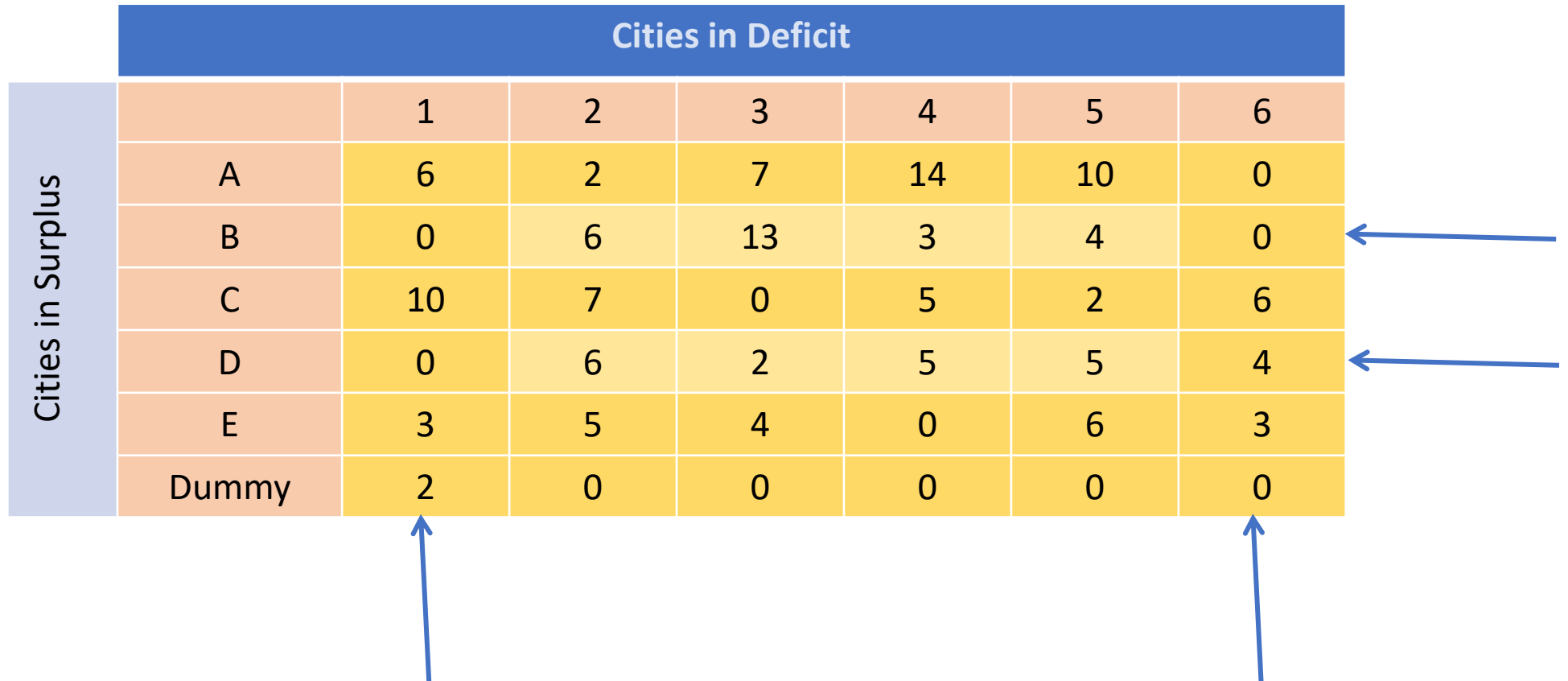
		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0

The diagram illustrates a transportation problem matrix. The columns are labeled 'Cities in Deficit' and numbered 1 through 6. The rows are labeled 'Cities in Surplus' and lettered A through E, plus a 'Dummy' row. The matrix contains numerical values representing supply and demand. Blue arrows point to specific cells: one arrow points to the cell (A, 6) which contains '0', another to (B, 6) which contains '0', a third to (D, 6) which contains '4', a fourth to (Dummy, 1) which contains '2', and a fifth to (Dummy, 6) which contains '0'. The cells containing '0' are highlighted in yellow, and the cells containing '4' and '0' in the Dummy row are highlighted in green.

ASSIGNMENT PROBLEMS

Drawing lines through marked columns and unmarked rows we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	2	7	14	10	0
	B	0	6	13	3	4	0
	C	10	7	0	5	2	6
	D	0	6	2	5	5	4
	E	3	5	4	0	6	3
	Dummy	2	0	0	0	0	0



ASSIGNMENT PROBLEMS

The smallest element in the unmarked cells is 2. Subtracting this element from cells untouched by lines and adding it to the cells at the cross section of two lines we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	8	2	7	14	10	2
	B	0	4	11	1	2	0
	C	10	7	0	5	2	6
	D	0	4	0	3	3	4
	E	5	5	4	0	6	5
	Dummy	4	0	0	0	0	2

ASSIGNMENT PROBLEMS

Subtracting smallest element in each row from all row elements and making allocations we get

		Cities in Deficit					
		1	2	3	4	5	6
Cities in Surplus	A	6	0	5	12	8	0
	B	0	4	11	1	2	0
	C	10	7	0	5	2	6
	D	0	4	0	3	3	4
	E	5	5	4	0	6	5
	Dummy	4	0	0	0	0	2

Therefore no truck is supplied to city 5. Truck goes from city A to 2, B to 6, C to 3, D to 1, E to 4 and F to 5.

Minimum distance travelled is $10+12+3+6+7=38$ kms

- TILL WE MEET AGAIN IN THE NEXT CLASS.....

