MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 15

Assignment Problems-Hungarian Method

ASSIGNMENT PROBLEMS

Example:

A manufacturing company decides to buy four subassemblies from four suppliers who can all supply each of the subassemblies. But the company policy is to buy one subassembly only from each supplier. Assign the different subassemblies to different suppliers so as to minimise cost.

		SUPPLIERS			
SUBASSEMBLIES		1	2	3	4
	1	15	13	14	17
	2	11	12	15	13
	3	13	12	10	11
	4	15	17	14	16

ASSIGNMENT PROBLEMS

Solution:

Subtracting the smallest element of each row from the elements of the corresponding row we get

SUBASSEMBLIES	SUPPLIERS 1	2	3	4
1	2	0	1	4
2	0	1	4	2
3	3	2	0	1
4	1	3	0	2

All columns do not contain zeroes. Therefore subtracting minimum column element from each element of the corresponding column we get

SUBASSEMBLIES	SUPPLIERS 1	2	3	4
1	2	0	1	3
2	0	1	4	1
3	3	2	0	0
4	1	3	0	1

Therefore assembly 2 is bought from supplier 1, 1 from 2, 4 from 3 and 3 from4

Total cost =13+11+11+14=49

ASSIGNMENT ALGORITHM

1. Develop the cost matrix for the given problem. If number of rows and columns are not equal, dummy rows and columns are used, with zero cost elements, to make the matrix a square matrix.
2. Develop the opportunity cost matrix by subtracting the smallest cost in each row from all the cells in the corresponding row and then by subtracting the minimum cost element of each column from all the cells of the corresponding column of the resultant matrix.
3. Make assignments as follows :
   i. Examine the rows one by one till a row with exactly one zero is obtained. Mark this zero as assigned and cancel all other zeros corresponding to the column of the assigned zero.
   ii. Repeat step 3(i) for each column with a single unmarked zero cancelling other zeroes in the corresponding row.
   iii. If a row and/or column has two or more unmarked zeroes and one can not be selected by inspection, then the zero should be assigned arbitrarily.
   iv. Continue the above process till all zeros are either assigned or struck off.

ASSIGNMENT ALGORITHM CONTINUED

4. If the number of assigned zeroes are equal to the number of rows or columns of the cost matrix, assignment is optimal. If not, proceed to step 5.
5. Revise the opportunity cost table obtained in step 3 as follows:

   Draw a set of horizontal and vertical lines to cover all the zeroes in the revised cost matrix by

i. Mark each row in which no assignment was made.
ii. Examine the marked rows to see if any zero is present in any cell of the row, if yes mark the corresponding columns(containing those zeroes).
iii. Examine the marked columns to see if any assigned zero occurs in these columns, if yes mark the corresponding rows with the assigned zeroes.
iv. Repeat the above process until no more rows or columns can be marked.
v. Draw straight lines through each marked column and unmarked row.

If the number of lines i.e. total assignments is equal to the number of rows or columns, the solution is optimal. Otherwise proceed to step 6.

ASSIGNMENT ALGORITHM CONTINUED

6. Prepare the new revised opportunity cost table.
   i. Choose the smallest element from the cells not covered by any line and subtract it from every element in the cells not covered by any line.
   ii. Add the smallest element from step 6(i) to the cells at which two lines intersect.
   iii. Elements covered by a single line remain unchanged.

7. Repeat steps 3 to 6 until an optimal solution is obtained

ASSIGNMENT PROBLEMS

Example 2:

A department has five employees with five jobs to be performed. The time (in hours) each man will take to perform each job is given in the effectiveness matrix below. How should the jobs be allocated, one per employee, so as to minimize the total man hours?

		Employees				
		1	2	3	4	5
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

ASSIGNMENT PROBLEMS

SOLUTION:

Subtracting the minimum row element from each cell of the corresponding row we get

		Employees				
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Since one unique job could not be assigned to each employee, marking rows without assignment

		Employees				
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Marking columns with zeros in marked row

		Employees				
		1	2	3	4	5
Jobs	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Marking rows with assigned zeros in marked columns

	Employees					
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Drawing lines through marked columns and unmarked rows Marking rows with assigned zeros in marked columns

		Employees				
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Solution is not optimal as 4 lines are there but rows=columns = 5

		Employees				
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	6	15	10	3
	C	8	5	0	0	0
	D	0	4	2	0	5
	E	3	5	6	0	8

ASSIGNMENT PROBLEMS

SOLUTION:

Subtracting the smallest element not covered by any line (2) and subtracting it from all the elements not covered by any line we get

	Employees					
Jobs		1	2	3	4	5
	A	7	0	8	10	11
	B	0	4	13	10	1
	C	8	5	0	0	0
	D	0	2	0	0	3
	E	3	3	4	0	6

ASSIGNMENT PROBLEMS

SOLUTION:

Adding the smallest element not covered by any line (2) to the cells at the intersection of two lines we get

		Employees				
		1	2	3	4	5
Jobs	A	9	0	8	12	11
	B	0	4	13	10	1
	C	10	5	0	2	0
	D	0	2	0	0	3
	E	3	3	4	0	6

ASSIGNMENT PROBLEMS

SOLUTION:

Repeating steps 3 to 6, the optimal solution is

Job	A	B	C	D	E
Employee	ii	I	v	iii	iv
Time	5	3	2	9	4

Total time -5+3+2+9+4=23 hours.

		Employees				
Jobs		1	2	3	4	5
	A	9	0	8	12	11
	B	0	4	13	10	1
	C	10	5	0	2	0
	D	0	2	0	0	3
	E	3	3	4	0	6

- TILL WE MEET AGAIN IN THE NEXT CLASS……….