MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 14

Transportation Problems-Unbalanced Profit maximization problems

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

Example:

A company has 3 factories manufacturing the same product and 5 sales agencies in different parts of the country. Production costs differ from factory to factory and the sales prices from agency to agency. The shipping cost per unit product from each factory to each agency is known. Given the following data, find the production and distribution schedules that will be most profitable for the company.

Factory		Production Cost/Unit (Rs.)	Maximum Capacity (No. of Units)
	1	18	140
	2	20	190
	3	16	115

Agency	1	2	3	4	5
Demand	74	94	69	39	119
Sales Price(Rs.)	35	37	36	39	34

Shipping Cost

Factory \ Agency	1	2	3	4	5
1	2	2	6	10	5
2	10	8	9	4	7
3	5	6	4	3	8

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

Profit/unit=sales price/unit – production cost/unit – transportation cost/unit

		Sales Agency					Production Capacity
Factory		1	2	3	4	5	
	1	35-18-2=15	37-18-2=17	36-18-6=12	39-18-10=11	34-18-5=11	140
	2	35-20-10=5	37-20-8=9	36-20-9=7	39-20-4=15	34-20-7=7	190
	3	35-16-5=14	37-16-6=15	36-16-4=16	39-16-3=20	34-16-8=10	115
Demand		74	94	69	39	119	395 445

Surplus capacity=445-395=50 units
Therefore Dummy Agency has 50 units demand with zero profit.

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

Maximization problem is converted to minimization problem by subtracting each profit from the highest profit.

	Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	15	17	12	11	11	0	140
	2	5	9	7	15	7	0	190
	3	14	15	16	20	10	0	115
Demand		74	94	69	39	119	50	

Subtracting each cell value from the highest value 20, the maximization problem is converted to a minimization problem .

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

- **Subtracting each cell value from the highest value 20, the maximization problem is converted to a minimization problem we get**

	Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	5	3	8	9	9	20	140 [2]
	2	15	11	13	5 39	13	20	190/151 [6]
	3	6	5	4	0	10	20	115 [4]
Demand		74 [1]	94 [2]	69 [4]	39/0 [5]	119 [1]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

	Sales Agency						Production Capacity
Factory	1	2	3	4	5	Dummy	
1	5	3	8	9	9	20	140 [2]
2	15	11	13	5 39	13	20	190/151 [6][2]
3	6	5	4 69	0	10	20	115/46 [4][1]
Demand	74 [1]	94 [2]	69/0 [4]	39/0 [5]	119 [1]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

	Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	5	3 94	8	9	9	20	140/46 [2][4]
	2	15	11	13	5 39	13	20	190/151 [6][2]
	3	6	5	4 69	0	10	20	115/46 [4][1][4]
Demand		74 [1]	94/0 [2]	69/0 [4]	39/0 [5]	119 [1]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

		Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	5 46	3 94	8	9	9	20	140/46/0 [2][4]
	2	15	11	13	5 39	13	20	190/151 [6][2]
	3	6	5	4 69	0	10	20	115/46/18 [4][1][4]
Demand		74/28 [1][9]	94/0 [2]	69/0 [4]	39/0 [5]	119 [1][3]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

		Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	5 46	3 94	8	9	9	20	140/46/0 [2][4]
	2	15	11	13	5 39	13	20	190/151 [6][2][7]
	3	6 28	5	4 69	0	10 18	20	115/46/18/0 [4][1][4][10]
Demand		74/28/0 [1][9]	94/0 [2]	69/0 [4]	39/0 [5]	119/101 [1][3]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

		Sales Agency						Production Capacity
Factory		1	2	3	4	5	Dummy	
	1	5 46	3 94	8	9	9	20	140/46/0 [2][4]
	2	15	11	13	5 39	13 101	20 50	190/151 [6][2][7]
	3	6 28	5	4 69	0	10 18	20	115/46/18/0 [4][1][4][10]
Demand		74/28/0 [1][9]	94/0 [2]	69/0 [4]	39/0 [5]	119/101 [1][3]	50 [0]	

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

- **Putting** $U_i + V_j$ = cost in cell(I,j) for all allocated cells we get

$U_1 + V_1 = 5$
$U_1 + V_2 = 3$
$U_2 + V_4 = 5$
$U_2 + V_5 = 13$
$U_2 + V_6 = 20$
$U_3 + V_1 = 6$
$U_3 + V_3 = 4$
$U_3 + V_5 = 10$

	V_1	V_2	V_3	V_4	V_5	V_6
U_1	5 46	3 94	8	9	9	20
U_2	15	11	13	5 39	13 101	20 50
U_3	6 28	5	4 69	0	10 18	20

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

- **Putting** $V_1 = 0$ we get from allocated cells

$U_1 + V_1 = 5$ or $U_1 + 0 = 5$ or $U_1 = 5$ (1)
$U_1 + V_2 = 3$ or $5 + V_2 = 3$ or $V_2 = -2$ (3)
$U_2 + V_4 = 5$ or $9 + V_4 = 5$ or $V_4 = -4$ (7)
$U_2 + V_5 = 13$ or $U_2 + 4 = 13$ or $U_2 = 9$ (6)
$U_2 + V_6 = 20$ or $9 + V_6 = 20$ or $V_6 = 11$ (8)
$U_3 + V_1 = 6$ or $U_3 + 0 = 6$ or $U_3 = 6$ (2)
$U_3 + V_3 = 4$ or $6 + V_3 = 4$ or $V_3 = -2$ (4)
$U_3 + V_5 = 10$ or $6 + V_5 = 10$ or $V_5 = 4$ (5)

	$V_1 = 0$	$V_2 = -2$	$V_3 = -2$	$V_4 = -4$	$V_5 = 4$	V_6
$U_1 = 5$	5 46	3 94	8	9	9	20
$U_2 = 9$	15	11	13	5 39	13 101	20 50
$U_3 = 6$	6 28	5	4 69	0	10 18	20

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

- **Considering unallocated cells and calculating** cost in cell(I,j) - $U_i + V_j$ for all unallocated cells we get

$U_1 + V_1 = 5$ or $U_1 + 0 = 5$ or $U_1 = 5$ (1)
$U_1 + V_2 = 3$ or $5 + V_2 = 3$ or $V_2 = -2$ (3)
$U_2 + V_4 = 5$ or $9 + V_4 = 5$ or $V_4 = -4$ (7)
$U_2 + V_5 = 13$ or $U_2 + 4 = 13$ or $U_2 = 9$ (6)
$U_2 + V_6 = 20$ or $9 + V_6 = 20$ or $V_6 = 11$ (8)
$U_3 + V_1 = 6$ or $U_3 + 0 = 6$ or $U_3 = 6$ (2)
$U_3 + V_3 = 4$ or $6 + V_3 = 4$ or $V_3 = -2$ (4)
$U_3 + V_5 = 10$ or $6 + V_5 = 10$ or $V_5 = 4$ (5)

	$V_1 = 0$	$V_2 = -2$	$V_3 = -2$	$V_4 = -4$	$V_5 = 4$	$V_6 = 11$
$U_1 = 5$	5 46	3 94	8-3=5	9-1=8	9-9=0	20-16=4
$U_2 = 9$	15-9=6	11-7=4	13-7=4	5 39	13 101	20 50
$U_3 = 6$	6 28	5-4=1	4 69	0-2=-2	10 18	20-17=3

Cell evaluation is negative for cell 3,4. Hence allocating 18 units to this cell we get

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

- **Cell evaluation is negative for cell 3,4. Hence allocating 18 units to this cell we get**

$U_1 + V_1 = 5$ or $U_1 + 0 = 5$ or $U_1 = 5$ (1)
$U_1 + V_2 = 3$ or $5 + V_2 = 3$ or $V_2 = -2$ (3)
$U_2 + V_4 = 5$ or $9 + V_4 = 5$ or $V_4 = -4$ (7)
$U_2 + V_5 = 13$ or $U_2 + 4 = 13$ or $U_2 = 9$ (6)
$U_2 + V_6 = 20$ or $9 + V_6 = 20$ or $V_6 = 11$ (8)
$U_3 + V_1 = 6$ or $U_3 + 0 = 6$ or $U_3 = 6$ (2)
$U_3 + V_3 = 4$ or $6 + V_3 = 4$ or $V_3 = -2$ (4)
$U_3 + V_5 = 10$ or $6 + V_5 = 10$ or $V_5 = 4$ (5)

	$V_1 = 0$	$V_2 = -2$	$V_3 = -2$	$V_4 = -4$	$V_5 = 4$	$V_6 = 11$
$U_1 = 5$	5 46	3 94	8-3=5	9-1=8	9-9=0	20-16=4
$U_2 = 9$	15-9=6	11-7=4	13-7=4	5 39-18	13 101+18	20 50
$U_3 = 6$	6 28	5-4=1	4 69	0-2=-2 +18	10 18-18	20-17=3

UNBALANCED TRANSPORTATION PROBLEM WITH PROFIT MAXIMIZATION

$U_1 + V_1 = 5$ or $U_1 + 0 = 5$ or $U_1 = 5$ (1)
$U_1 + V_2 = 3$ or $5 + V_2 = 3$ or $V_2 = -2$ (3)
$U_2 + V_4 = 5$ or $U_2 - 6 = 5$ or $U_2 = 11$ (6)
$U_2 + V_5 = 13$ or $11 + V_5 = 13$ or $V_5 = 2$ (7)
$U_2 + V_6 = 20$ or $11 + V_6 = 20$ or $V_6 = 9$ (8)
$U_3 + V_1 = 6$ or $U_3 + 0 = 6$ or $U_3 = 6$ (2)
$U_3 + V_3 = 4$ or $6 + V_3 = 4$ or $V_3 = -2$ (4)
$U_3 + V_4 = 0$ or $6 + V_4 = 0$ or $V_4 = -6$ (5)

	$V_1 = 0$	$V_2 = -2$	$V_3 = -2$	$V_4 = -6$	$V_5 = 2$	$V_6 = 9$
$U_1 = 5$	5 46	3 94	8-3=5	9+1=10	9-7=2	20-14=6
$U_2 = 11$	15-11=4	11-9=2	13-9=4	5 21	13 119	20 50
$U_3 = 6$	6 28	5-4=1	4 69	0 18	10-8=2	20-15=5

- **All cell allocations are positive. Hence allocation is optimal.**
- **Z_{max} = 15X46+17X94+15X21+7X119+0X50+14X28+16X69+20X18= Rs. 5,292/-**

- TILL WE MEET AGAIN IN THE NEXT CLASS.....