MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 13

Transportation Problems-Optimal solutions

TRANSPORTATION MODEL

Example:

A company has four warehouses and six stores. The warehouses altogether have a surplus of 22 units of a given commodity divided among them as follows:

Warehouses	1	2	3	4
Surplus	5	6	2	9

The six stores altogether need 22 units of the commodity. Individual requirements at stores 1, 2, 3, 4, 5 and 6 are 4, 4, 6, 2, 4 and 2 units respectively.

Cost of shipping one unit of commodity from warehouse I to store j in rupees is given in the matrix below:

		STORES					
WAREHOUSES		1	2	3	4	5	6
	1	9	12	9	6	9	10
	2	7	3	7	7	5	5
	3	6	5	9	11	3	11
	4	6	8	11	2	2	10

How should the products be shipped from the warehouses to the stores so that the transportation cost is minimized.

OPTIMALITY TEST

Method :

1. Perform optimality test to check whether the feasible solution obtained is optimum.
2. To perform optimality test successfully
   i. Number of allocations should be row+column-1
   ii. Allocated row+column-1 cells should occupy independent positions i.e.it is not possible to travel from an allocated cell back to itself by a series of horizontal and vertical jumps from one occupied cell to another without a reversal of the same route
3. Each vacant unallocated cell is tested to see whether an allocation in it will reduce the total transportation cost. The two methods used are **Stepping Stone Method** and the **Modified Distribution(MODI) method**.

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

Degeneracy in transportation Problem: To perform optimality test row+column-1(4+6-1) allocations should be there. But here we have 8 allocations.

		STORES					
		1	2	3	4	5	6
WAREHOUSES	1	9	12	9 5	6	9	10
	2	7	3 4	7	7	5	5 2
	3	6 1	5	9 1	11	3 ε	11
	4	6 3	8	11	2 2	2 4	10

Unoccupied cell(3,5) has least cost Rs. 3/- But allocation in this cell leads to a closed loop and degeneracy cannot be removed.

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

The next lowest cost empty cells are cells (2,5) and (3,2).

None of these cells form closed loops

		STORES					
WAREHOUSES		1	2	3	4	5	6
	1	9	12	9 5	6	9	10
	2	7	3 4	7	7	5 ε	5 2
	3	6 1	5	9 1	11	3	11
	4	6 3	8	11	2 2	2 4	10

Now degeneracy is removed as allocation is 9(6+4-1)

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

Stepping Stone Method: The net change in the transportation cost as a result of the changes occurring on allotting one unit to an empty cell is called the **EVALUATION of the cell** under consideration**.**

		STORES					
WAREHOUSES		1	2	3	4	5	6
	1	9 +1	12	9 5 -1	6	9	10
	2	7	3 4	7	7	5 ε	5 2
	3	6 1 -1	5	9 1 +1	11	3	11
	4	6 3	8	11	2 2	2 4	10

Suppose we allot one unit to cell (1,1), we must deduct from other cells of the row and column to meet demand and supply constraints.

Change in cost =+9-9+9-6=3

A positive evaluation for a cell indicates increase in cost if allocation is made in the cell. Allocation in cell (1,1) increases cost by 3 units per allocation and hence is not advised.

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

Allocating one unit in cell (2,3), change in cost is +7-5+2-6+6-9=-5.

Hence allocation in cell(2,3) will reduce the cost

		STORES					
WAREHOUSES		1	2	3	4	5	6
	1	9	12	9 5	6	9	10
	2	7	3 4	7 +1	7	5 ε -1	5 2
	3	6 1 +1	5	9 1 -1	11	3	11
	4	6 3 -1	8	11	2 2	2 4 +1	10

Stepping stone method evaluates one cell at a time to decide on the best allocation

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

Modified Distribution method(MODI) or U-V method:

Here cell evaluations of all the unoccupied cells are calculated simultaneously and a single closed path is traced for the cell for which allocations have been made.

	V_1	V_2	V_3	V_4	V_5	V_6
U_1	9	12	9 5	6	9	10
U_2	7	3 4	7	7	5 ε	5 2
U_3	6 1	5	9 1	11	3	11
U_4	6 3	8	11	2 2	2 4	10

Step 1: Write down a cost matrix only with the cells for which allocations have already been made.

Step 2: Let there be a set of numbers U_i associated with each row and V_j associated with each column such that $U_i + V_j$ = cost in cell(I,j)

Transportation Problem-Finding an Optimal Solution

Modified Distribution method(MODI) or U-V method:

In this problem considering the allocated cells and putting V_1=0:

$U_1 + V_3 = 9$

$U_2 + V_2 = 3$

$U_2 + V_5 = 5$

$U_2 + V_6 = 5$

$U_3 + V_1 = 6$

$U_3 + V_3 = 9$

$U_4 + V_1 = 6$

$U_4 + V_4 = 2$

$U_4 + V_5 = 2$

	V_1	V_2	V_3	V_4	V_5	V_6
U_1	9	12	9 (5)	6	9	10
U_2	7	3 (4)	7	7	5 (ε)	5 (2)
U_3	6 (1)	5	9 (1)	11	3	11
U_4	6 (3)	8	11	2 (2)	2 (4)	10

Step 1: Write down a cost matrix only with the cells for which allocations have already been made.

Step 2: Let there be a set of numbers U_i associated with each row and V_j associated with each column such that $U_i + V_j$ = cost in cell(I,j)

Modified Distribution method(MODI) or U-V method:

In this problem considering the allocated cells and putting V_1=0:

$U_1 + V_3$ =9 or U_1 + 3 =9 or U_1 = 6 (8)

$U_2 + V_2$ =3 or 9 + V_2 =3 or V_2 = -6 (9)

$U_2 + V_5$ =5 or U_2 - 4 =5 or U_2 = 9 (4)

$U_2 + V_6$ =5 or 9 + V_6 =5 or V_6 = -4 (5)

$U_3 + V_1$ =6 or U_3+ 0 =6 or U_3 = 6 (6)

$U_3 + V_3$ =9 or 6 + V_3 =9 or V_3 =3 (7)

$U_4 + V_1$ =6 or U_4 + 0 =6 or U_4 =6 (1)

$U_4 + V_4$ =2 or 6 + V_4 =2 or V_4 = -4 (3)

$U_4 + V_5$ =2 or 6 + V_5 =2 or $\mathbf{V_5}$ **=** -4 (2)

	V_1 =0	V_2=-6	V_3 =3	V_4 =-4	V_5 =-4	V_6 =-4
U_1=6	9 – (6+0)=3	12–(6-6)=12	9 5	6-(6-2)=4	9-(6-4)=7	10-(6-4)=8
U_2=9	7-(9-0)= -2	3 4	7-(9+3)=-5	7-((9-4)=2	5 ε	5 2
U_3=6	6 1	5-(6-6)=5	9 1	11-(6-4)=9	3-(6-4)=1	11-(6-4)=9
U_4=6	6 3	8-(6-6)=8	11-(6+3)=2	2 2	2 4	10-(6-4)=8

Step 3: Fill the vacant cells with the sum of U_i and V_j

Step 4: Subtract the cell values obtained in the previous step from the original cost matrix. The resultant matrix is called the **CELL EVALUATION MATRIX.**

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

Step 5: If any of the cell evaluations is negative, the basic feasible solution is not optimal. Here two cwll evaluations are negative.

Step 6: Iterate towards an optimal solution.

i. **Find the most negative cell evaluation from the cell evaluation matrix. If a tie occurs, choose anyone.**

ii. **Copy the initial feasible solution marking the empty cell with most negative cell evaluation. This cell is called the identified cell. Here cell(2,3) is the identified cell.**

iii **Trace a path beginning with the identified cell passing horizontally and vertically with corners on allocated cells. The path may skip over any number of occupied or vacant cells.**

	V_1 =0	V_2=-6	V_3 =3	V_4 =-4	V_5 =-4	V_6 =-4
U_1=6	9 – (6+0)=3	12–(6-6)=12	9 5	6-(6-2)=4	9-(6-4)=7	10-(6-4)=8
U_2=9	7-(9-0)= -2	3 4	7-(9+3)=-5	7-((9-4)=2	5 ε	5 2
U_3=6	6 1	5-(6-6)=5	9 1	11-(6-4)=9	3-(6-4)=1	11-(6-4)=9
U_4=6	6 3	8-(6-6)=8	11-(6+3)=2	2 2	2 4	10-(6-4)=8

TRANSPORTATION PROBLEM-FINDING AN OPTIMAL SOLUTION

iv. **Mark the identified cell as +ve and each occupied cell at the corner as –ve and +ve alternately.**

v. **The smallest allocation in a –ve marked cell is allocated to the identified cell. This allocation is added and subtracted from the corner cells maintaining row and column balance.**

Step 6: Check for optimality. Repeat steps 3 to 4 till no more cell evaluations are negative.

Cell (2,3) has the most negative cell e
improve the solution

	$V_1=0$	$V_2=-6$	$V_3=3$	$V_4=-4$	$V_5=-4$	$V_6=-4$
$U_1=6$	9	12	9 / 5	6	9	10
$U_2=9$	7	3 / 4	7 / + ε	7	5 / ε-ε	5 / 2
$U_3=6$	6 / 1+ε	5	9 / 1-ε	11	3	11
$U_4=6$	6 / 3-ε	8	11	2 / 2	2 / 4+ε	10

- TILL WE MEET AGAIN IN THE NEXT CLASS……….