

MB106

QUANTITATIVE TECHNIQUES



**OPERATION
RESEARCH**

MODULE I

LECTURE 10

Duality and it's Economic Interpretation

SHADOW PRICE AND ECONOMIC INTERPRETATION

Example:

A company makes three products X, Y and Z out of three raw materials A, B and C. The number of units of raw materials required to produce one unit of the product is as given in the table below:

	X	Y	Z
A	1	2	1
B	2	1	4
C	2	5	1

The unit profit contribution of the products X, Y and Z are Rs. 40, Rs 25 and Rs 50 respectively and the number of units of raw materials available are 36, 60 and 45 respectively.

- (i) Determine the product mix that will maximize the total profit.
- (ii) From the final simplex table, write the solution to the dual and

SHADOW PRICE AND ECONOMIC INTERPRETATION

Let x_1 be the number of units of X produced

Let x_2 be the number of units of Y produced

Let x_3 be the number of units of Z produced

Therefore the LPP is

$$\text{Maximize } Z = 40x_1 + 25x_2 + 50x_3$$

Subject to

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \rightarrow \text{non negativity restrictions}$$

$$x_1 + 2x_2 + x_3 \leq 36 \rightarrow (1)$$

$$2x_1 + x_2 + 4x_3 \leq 60 \rightarrow (2)$$

$$2x_1 + 5x_2 + x_3 \leq 45 \rightarrow (3)$$

SHADOW PRICE AND ECONOMIC INTERPRETATION

Introducing slack variables s_1 , s_2 and s_3 we get

$$\text{Maximize } Z = 40x_1 + 25x_2 + 50x_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0 \rightarrow \text{non negativity restrictions}$$

$$x_1 + 2x_2 + x_3 + s_1 = 36 \rightarrow (1)$$

$$2x_1 + x_2 + 4x_3 + s_2 = 60 \rightarrow (2)$$

$$2x_1 + 5x_2 + x_3 + s_3 = 45 \rightarrow (3)$$

Putting $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ we get

$$s_1 = 36, s_2 = 60, s_3 = 45$$

SHADOW PRICE AND ECONOMIC INTERPRETATION

Objective function →	C_j	40	25	50	0	0	0		
e_i	CSV	x_1	x_2	x_3	s_1	s_2	s_3	b_i	θ
0	s_1	1	2	1	1	0	0	36	36
0	s_2	2	1	4	0	1	0	60	15
0	s_3	2	5	1	0	0	1	45	45
$Z_j = e_i a_{ij}$		0	0	0	0	0	0	0	
$C_j - Z_j$		40	25	50	0	0	0		

KEY
ROW

← outgoing variable s_2

KEY COLUMN

incoming variable x_3

KEY ELEMENT

SHADOW PRICE AND ECONOMIC INTERPRETATION

Objective function →	C_j	40	25	50	0	0	0		
e_i	CSV	x_1	x_2	x_3	s_1	s_2	s_3	b_i	θ
0	s_1	1/2	7/4	0	1	-1/4	0	21	42
50	x_3	1/2	1/4	1	0	1/4	0	15	30
0	s_3	3/2	19/4	0	0	-1/4	1	30	20
$Z_j = e_i a_{ij}$		25	25/2	50	0	25/2	0	750	
$C_j - Z_j$		15	25/2	0	0	-25/2	0		

KEY
ROW

← outgoing variable s_3

KEY COLUMN

incoming variable x_1

KEY ELEMENT

SHADOW PRICE AND ECONOMIC INTERPRETATION

Objective function →	C_j	40	25	50	0	0	0	
e_i	CSV	x_1	x_2	x_3	s_1	s_2	s_3	b_i
0	s_1	0	1/6	0	1	-1/6	-1/3	11
50	x_3	0	-4/3	1	0	1/3	-1/3	5
40	x_1	1	19/6	0	0	-1/6	2/3	20
$Z_j = e_i a_{ij}$		40	60	50	0	10	10	1050
$C_j - Z_j$		0	-35	0	0	-10	-10	

therefore the optimal solution to the primal is

$$Z_{\max} = Rs\ 1050, \quad x_1 = 20, \quad x_2 = 0, \quad x_3 = 5$$

Solution to the dual

$$W_{\min} = Rs\ 1050, \quad = 0, \quad = 10, \quad = 10 \text{ (} Z_j \text{ values from optimal simplex tableau)}$$

SHADOW PRICE AND ECONOMIC INTERPRETATION

The dual problem is

$$\text{Minimize } W = 36y_1 + 60y_2 + 45y_3$$

Subject to

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \rightarrow \text{non negativity restrictions}$$

$$y_1 + 2y_2 + 2y_3 \geq 40 \rightarrow (1)$$

$$2y_1 + y_2 + 5y_3 \geq 25 \rightarrow (2)$$

$$y_1 + 4y_2 + y_3 \geq 50 \rightarrow (3)$$

ECONOMIC INTERPRETATION

- ❖ The marginal values of raw materials A, B and C are Rs 0, Rs 10 and Rs 10 per unit respectively when sold instead of using in-house.
- ❖ They represent the accounting values and not necessarily the market price.
- ❖ The purchaser will try to minimize his purchase cost which will be Rs 1050/-
- ❖ Same contribution of Rs 1050/- will be obtained by the organization irrespective of whether the raw materials are used in-house or sold out.

SHADOW PRICE

- ❖ The net evaluation row or the index row ($C_j - Z_j$) indicates the amount by which the profit will increase if a unit of the corresponding variable is introduced into the solution in case of positive values.
- ❖ A negative value indicates loss.
- ❖ The net evaluation row in case of slack variables indicates the shadow prices which are the accounting values for the resources. (-10 under s_2 indicates a loss of Rs 10/- for every unit of B unutilized.

- TILL WE MEET AGAIN IN THE NEXT CLASS.....

