MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 6

Linear Programming: Big M Method

LPP-Big M Method Application

- This method is suitable for problems with greater than equal to or equal to constraints.
- Start by expressing the linear programming problem in the standard form.
- Add non negative variables to the left hand side of all the constraints of = or ≥ type called ***artificial variables***.
- The artificial variables violate the equality constraints and are hence removed after obtaining the initial basic feasible solution.
- Proceed with the regular simplex method.

LPP-Big M Method Application

Example:

Maximize $Z=3\,x_1 - x_2$

Subject to

$x_1 \geq 0, x_2 \geq 0$ → non negativity restrictions

$2\,x_1 + x_2 \geq 2$ →(1)

$x_1+3x_2 \leq 3$ → (2)

$x_2 \leq 4$ →(3)

LPP-BIG M METHOD APPLICATION

❖**Introducing slack variables to convert inequalities into equalities**

Maximize $Z=3x_1 – x_2 +0s_1 +0s_2 +0s_3$

Subject to

$2 x_1+x_2- s_1 = 2$ → (1)

$x_1+3x_2+s_2 =3$ → (2)

$x_2+s_3 = 4$ → (3)

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0 , s_2 \geq 0, s_3 \geq 0$ → non negativity restrictions

Putting $x_1=0$ and $x_2=0$ in constraints 1, 2 and 3 we get

$s_1 = -2, s_2 = 3, s_3 = 4$

LPP-BIG M METHOD APPLICATION

- Since negative values for the slack variables are not allowed we introduce artificial variable A.

$2x_1 + x_2 - s_1 + A = 2 \rightarrow (1)$

$x_1 + 3x_2 + s_2 = 3 \rightarrow (2)$

$x_2 + s_3 = 4 \rightarrow (3)$

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0, A \geq 0 \rightarrow$ non negativity restrictions

Putting $x_1=0$, $x_2=0$ and $s_1=0$ in constraints 1, 2 and 3 we get

$A = 2, s_2 = 3, s_3 = 4$

To eliminate artificial variables from the final solution (because artificial variables with values greater than zero destroy the equality required by the LP Model), a large *penalty* or negative value(-M) is assigned to the artificial variable in the objective function

Maximize $Z = 3x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - MA$

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	3	-1	0	0	0	-M		
e_i		x_1	x_2	s_1	s_2	s_3	A	b_i	θ
-M	***A***	2	1	-1	0	0	1	2	1
0	s_2	1	3	0	1	0	0	3	3
0	s_3	0	1	0	0	1	0	4	∞
$Z_j=e_ia_{ij}$		-2M	-M	M	0	0	-M		
C_j- Z_j		3+2M	-1+M	-M	0	0	0		

KEY ROW

outgoing variable **A**

KEY COLUMN

KEY ELEMENT

incoming variable X_1

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	3	-1	0	0	0	-M		
e_i		x_1	x_2	s_1	s_2	s_3	A	b_i	ϴ
3	x_1	1	1/2	-1/2	0	0	1/2	1	
0	s_2	1	3	0	1	0	0	3	
0	s_3	0	1	0	0	1	0	4	
$Z_j=e_ia_{ij}$									
$C_j- Z_j$									

Dividing the key row by 2 to get unity for key element

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	3	-1	0	0	0		
e_i		x_1	x_2	s_1	s_2	s_3	b_i	θ
3	x_1	1	1/2	-1/2	0	0	1	-2
0	s_2	0	5/2	1/2	1	0	2	4
0	s_3	0	1	0	0	1	4	∞
$Z_j = e_i a_{ij}$		3	3/2	-3/2	0	0		
$C_j - Z_j$		0	-5/2	3/2	0	0		

KEY ROW

outgoing variable S_2

KEY COLUMN

KEY ELEMENT

incoming variable S_1

Subtracting the key row from row 2 to get unity for key element and ZERO for other elements of key column

(Since artificial variable A has been thrown out of the solution, the column representing A is deleted.)

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	3	-1	0	0	0		
e_i		x_1	x_2	s_1	s_2	s_3	b_i	θ
3	x_1	1	1/2	-1/2	0	0	1	-2
0	s_1	0	5	1	2	0	4	4
0	s_3	0	1	0	0	1	4	∞
$Z_j=e_ia_{ij}$		3	3/2	-3/2	0	0		
C_j- Z_j		0	-5/2	3/2	0	0		

KEY ROW

outgoing variable S_2

KEY COLUMN

KEY ELEMENT

incoming variable S_1

Multiplying row 2 by 2 to make the key element 1

LPP-BIG M METHOD APPLICATION

Objective function →	C_j	3	-1	0	0	0		
e_i		x_1	x_2	s_1	s_2	s_3	b_i	θ
3	x_1	1	3	0	1	0	3	
0	s_1	0	5	1	2	0	4	
0	s_3	0	1	0	0	1	4	
$Z_j=e_ia_{ij}$		3	9	0	3	0		
C_j- Z_j		0	-10	0	-3	0		

Dividing row 2 by 2 and adding to row 1 to make all elements in key column, other than key element, ***ZERO.***

All C_j- Z_j s are negative or zero. Hence no further improvement of the solution is possible.

Therefore $x_1=3$, $x_2=0$, $s_1=4$, $s_2=0$, $s_3=4$

$$Z_{max}=3X3-0+0X4+0X0+0X4=9$$

- TILL WE MEET AGAIN IN THE NEXT CLASS……….