MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 5

Linear Programming: Simplex Method

LPP-Simplex Method Application

- When number of decision variables exceeds two.
- When number of decision variables exceed number of constraints.
- When the objective function as well as constraints contain linear relations in terms of decision variables(equations or inequations)

LPP-SIMPLEX METHOD

Example : Three grades of coal A, B and C contain phosphorus and ash as impurities. In a particular industrial process, fuel upto 100 tons(maximum) is required which should contain ash not more than 3% and phosphorus not more than 0.03%. It is desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of grades are given below.

Coal	Phosphorus	Ash	Profits in Rs./ton
A	0.02	2.0	12.00
B	0.04	3.0	15.00
C	0.03	5.0	14.00

Find the proportions in which the three grades be used

LPP-SIMPLEX METHOD

Decision Variables:

Let x_1 be the proportion of grade A coal

Let x_2 be the proportion of grade B coal

Let x_3 be the proportion of grade C coal

Objective function:

Maximize $Z=12\ x_1 +15\ x_2 +14\ x_3$

Subject to

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \rightarrow$ non negativity restrictions

$0.02\ x_1+0.04\ x_2 +0.03\ x_3 \leq 0.03(x_1+ x_2 + x_3) \rightarrow (1)$

Or $2x_1+4\ x_2 +3\ x_3 \leq 3(x_1+ x_2 + x_3)$

Or $-x_1 + x_2 \leq 0 \rightarrow (1)$

LPP-SIMPLEX METHOD

$2.0x_1+3.0x_2+5.0x_3 \leq 3(x_1+x_2+x_3) \rightarrow (2)$

Or $2x_1+3x_2+5x_3 \leq 3(x_1+x_2+x_3)$

Or $-x_1+2x_3 \leq 0 \rightarrow (2)$

$x_1+x_2+x_3 \leq 100 \rightarrow (3)$

Thus on introducing slack variables and converting inequalities to equalities the problem becomes:

Maximize $Z=12x_1+15x_2+14x_3+0s_1+0s_2+0s_3$

Subject to

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \rightarrow$ non negativity restrictions

$-x_1+x_2+s_1=0 \rightarrow (1)$

$-x_1+2x_3+s_2=0 \rightarrow (2)$

$x_1+x_2+x_3+s_3=100 \rightarrow (3)$

LPP-SIMPLEX METHOD

Let m be the number of equations

Let n be the number of constraints.

We can solve only for m variables.

Therefore let m variables be the CURRENT SOLUTION VARIABLES(Basic Variables)

Therefore m-n variables are non-basic variables equated to zero.

Here $Z=12x_1+15x_2+14x_3+0s_1+0s_2+0s_3$

Number of equations=3

Number of variables=6

Hence basic variables=3

Non-basic variables=6-3=3

Let x_1, x_2, x_3 be the non-basic variables equated to zero.

Therefore $x_1 = x_2 = x_3 = 0$

LPP-SIMPLEX METHOD

Therefore substituting x_1=0, x_2 =0, x_3 =0 in equations 1, 2, and 3 we get

0 + 0 +s_1=0 or s_1=0 →(1)

0 + 0 +s_2=0 or s_2=0 →(2)

0+ 0 + 0 +s_3=100 or s_3=100 →(3)

e_i →coefficient of the basic variable in the objective function

Objective function→	c_j	12	15	14	0	0	0		
e_i	CSV	x_1	x_2	X_3	s_1	s_2	s_3	b_i	
0	s_1	-1	1	0	1	0	0	0(ἐ)	
0	s_2	-1	0	2	0	1	0	0(ἐ)	
0	s_3	1	1	1	0	0	1	100	
$Z_j=e_ia_{ij}$		0	0	0	0	0	0	Profit lost per ton	
C_j- Z_j		12	15	14	0	0	0	Net profit per ton	

LPP-SIMPLEX METHOD

- ***OPTIMALITY TEST***:- A positive C_j- Z_j indicates scope for improvement of the current feasible solution in case of a maximization problem.
- Positive C_j- Z_j shows scope for profit improvement while negative C_j- Z_j indicates decrease in profit.
- Column with maximum C_j- Z_j is the key column(K).

Objective function→	C_j	12	15	14	0	0	0		
e_i	CSV	x_1	x_2	X_3	s_1	s_2	s_3	b_i	
0	s_1	-1	1	0	1	0	0	0(ε)	
0	s_2	-1	0	2	0	1	0	0(ε)	
0	s_3	1	1	1	0	0	1	100	
$Z_j=e_ia_{ij}$		0	0	0	0	0	0	Profit lost per ton	
C_j- Z_j		12	15	14	0	0	0	Net profit per ton	

LPP-SIMPLEX METHOD

- The variable representing the key column enters the solution here x_2.
- Divide elements under column b_i by corresponding elements of column K. giving column Θ.
- Row containing minimum value(positive ratio) in column is the key row representing the outgoing variable here s_1
- The element at the point of intersection of key row and key column is the pivot or key element .

Objective function→	c_j	12	15	14	0	0	0		
e_i	CSV	x_1	x_2	x_3	s_1	s_2	s_3	b_i	Θ
0	s_1	-1	1	0	1	0	0	ε	ε
0	s_2	-1	0	2	0	1	0	ε	∞
0	s_3	1	1	1	0	0	1	100	100
$Z_j=e_i a_{ij}$		0	0	0	0	0	0	Profit lost per ton	
$C_j- Z_j$		12	15	14	0	0	0	Net profit per ton	

LPP-SIMPLEX METHOD

- Divide/multiply the key row by a common factor to make the key element UNITY.
- Now all the elements in column K are made ***ZERO*** except the key element which is ***ONE*** by subtracting or adding the proper multiples of the key row to other rows.
- Now column of x_1 contains highest positive value of $C_j - Z_j$ and hence should be the key column.
- Now dividing the elements of column b_i by the corresponding elements of the **key column** we get the new column ϴ.
- Row of s_3 contains the minimum positive ratio ϴ and hence is the outgoing variable(KEY ROW)

KEY ELEMENT

Objective function→	C_j	**12**	**15**	**14**	**0** ε	**0**	**0**		
e_i	CSV	x_1	x_2	X_3	s_1	s_2	s_3	b_i	ϴ
15	x_2	-1	1	0	1	0	0	ε	-ε
0	s_2	-1	0	2	0	1	0	ε	-ε
0	s_3	2	0	1	-1	0	1	100-ε	50-ε/2
$Z_j=e_i a_{ij}$		-15	15	0	15	0	0	Profit lost per ton	
$C_j - Z_j$		27	0	14	-15	0	0	Net profit per ton	

LPP-SIMPLEX METHOD

- ❖ Divide/multiply the key row by a common factor to make the key element UNITY.
- ❖ Now all the elements in column K are made ***ZERO*** except the key element which is ***ONE*** by subtracting or adding the proper multiples of the key row to other rows.
- ❖ Column of X_3 contains highest positive value of C_j- Z_j and hence should be the key column.
- ❖ Now dividing the elements of column b_i by the corresponding elements of the key column we the new column ϴ. ***S_2*** has lowest positive ratio in ϴ and hence becomes the key row. KEY ELEMENT

Objective function→	C_j	12	15	14	0	0	0		
e_i	CSV	x_1	x_2	X_3	ε s_1	s_2	s_3	b_i	ϴ
15	x_2	0	1	1/2	1/2	0	1/2	50+ε/2	100+ε
0	s_2	0	0	5/2	-1/2	1	1/2	50+ε/2	20+ε/5
12	x_1	1	0	1/2	-1/2	0	1/2	50-ε/2	100-ε
$Z_j=e_ia_{ij}$		12	15	27/2	3/2	0	27/2	Profit lost per ton	
C_j- Z_j		0	0	1/2	-3/2	0	-27/2	Net profit per ton	

LPP-SIMPLEX METHOD

- Divide/multiply the key row by a common factor to make the key element UNITY.
- Now all the elements in column K are made ***ZERO*** except the key element which is ***ONE*** by subtracting or adding the proper multiples of the key row to other rows.
- Now all C_j- Z_j s are zero or negative leaving no scope for improvement.
- Here x_1=40-3ε/5, x_2=40+2ε/5 and X_3=20+ε/5. neglecting ε we get

x_1 =40, , x_2 =40, X_3 =20 and Z=12X40+15X40+14X20=Rs. 1360/- (***OPTIMUM SOLUTION***)

Objective function→	C_j	12	15	14 ε	0	0	0	
e_i	CSV	x_1	x_2	X_3	s_1	s_2	s_3	b_i
15	x_2	0	1	0	3/5	-1/5	2/5	40+2ε/5
14	X_3	0	0	1	-1/5	2/5	1/5	20+ε/5
12	x_1	1	0	0	-2/5	-1/5	2/5	40-3ε/5
$Z_j=e_ia_{ij}$		12	15	14	7/5	1/5	68/5	
C_j- Z_j		0	0	0	-7/5	-1/5	-68/5	

LPP-SIMPLEX RULES

- FEASIBILITY CONDITION-The leaving or outgoing variable is the basic variable corresponding to the minimum positive ratio obtained by dividing the elements of column b by the corresponding elements of the key column.
- OPTIMALITY CONDITION-The entering(incoming) variable is the non basic variable corresponding to the maximum positive(maximum negative) value of C_j- Z_j in a maximization(minimization) problem.

- TILL WE MEET AGAIN IN THE NEXT CLASS……….