MB106
QUANTITATIVE TECHNIQUES

MODULE I

LECTURE 4

Linear Programming: Graphical solution continued

GRAPHICAL SOLUTION OF TWO-VARIABLE L.P. PROBLEM - AN INDEFINITE NUMBER OF OPTIMAL SOLUTIONS

Example: A firm uses lathes, milling machines and grinding machines to produce two machine parts. Given below are the machine times required for each part, the machining times available on different machines and the profit on each machine part

Type of machine	Machining time required I	II	Maximum time available per week in mins
Lathes	12	6	3000
Milling machines	4	10	2000
Grinding machines	2	3	900
Profit/unit	Rs. 40	Rs. 100	

Find the number of parts I and II to be manufactured per week to maximize the profit.

Graphical Solution of Two-Variable L.P. Problem - An Indefinite Number of Optimal Solutions

Decision Variables: Let x_1 be the number of parts I manufactured

Let x_2 be the number of parts II manufactured

Objective function: Maximize $Z=40\ x_1 +100\ x_2$

Slope Calculation $40\ x_1 +100\ x_2 =0$ or $x_1 = -2\ x_2 /5$ or $x_1 / x_2 = -2/5$

Constraints: $12\ x_1 +6\ x_2 \le 3000$

$4\ x_1 +10\ x_2 \le 2000$

$2\ x_1 +3\ x_2 \le 900$

$x_1 \ge 0,\ x_2 \ge 0$

GRAPHICAL SOLUTION OF TWO-VARIABLE L.P. PROBLEM - AN INDEFINITE NUMBER OF OPTIMAL SOLUTIONS

For plotting the graph, identifying two points on each line :-

$12\ x_1 + 6\ x_2 = 3000$

Putting $x_1 = 0$ we get $x_2 = 3000/6 = 500$ and putting $x_2 = 0$ we get $x_1 = 3000/12 = 250$

Therefore the two points on the first line are **(0,500) and (250,0)**

$4\ x_1 + 10\ x_2 = 2000$

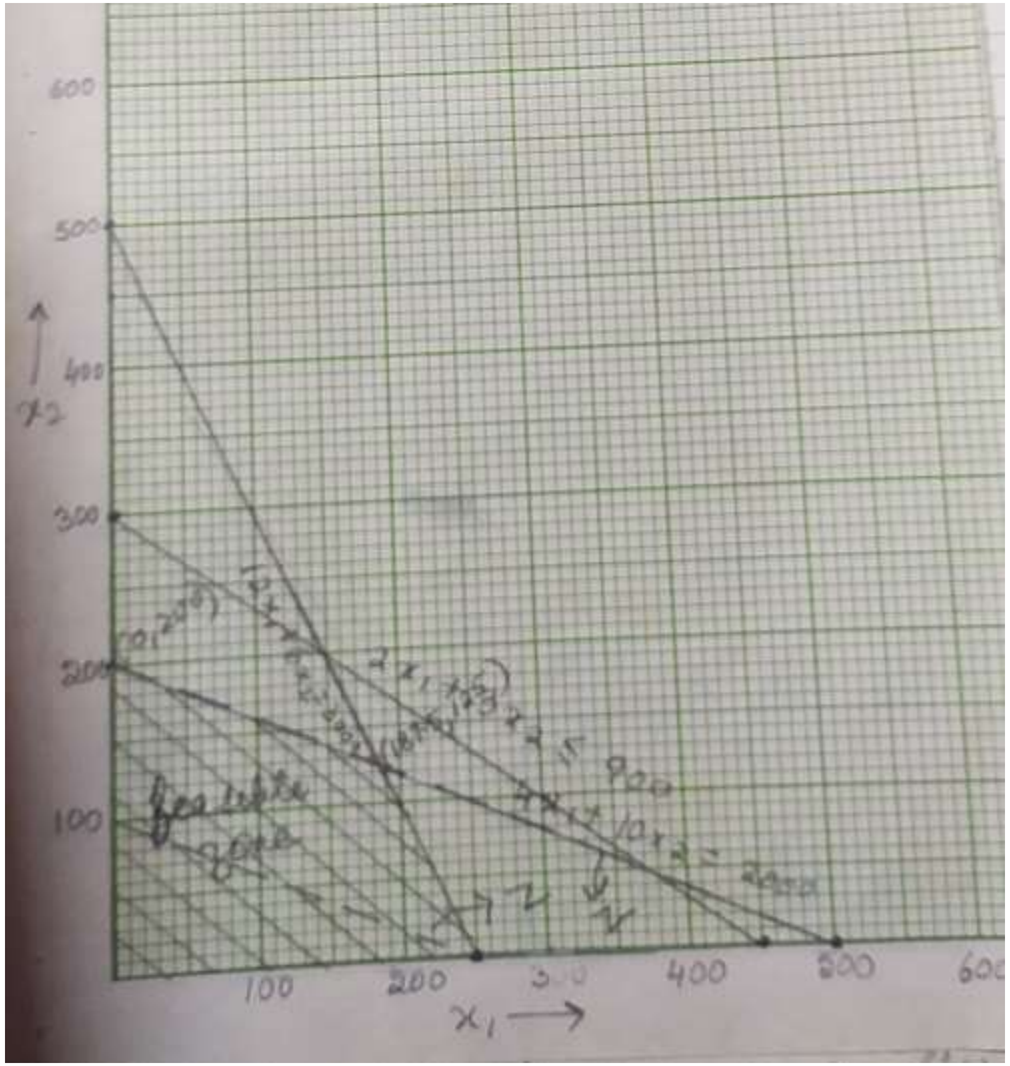
Putting $x_1 = 0$ we get $x_2 = 2000/10 = 200$ and putting $x_2 = 0$ we get $x_1 = 2000/4 = 500$

Therefore the two points on the first line are **(0,200) and (500,0)**

$2\ x_1 + 3\ x_2 = 900$

Putting $x_1 = 0$ we get $x_2 = 900/3 = 300$ and putting $x_2 = 0$ we get $x_1 = 900/2 = 450$

Therefore the two points on the first line are **(0,300) and (450,0)**



Graphical Solution of Two-Variable L.P. Problem - An Indefinite Number of Optimal Solutions

The constraints $4\ x_1 + 10\ x_2 = 2000 \rightarrow 1$

and $12\ x_1 + 6\ x_2 = 3000 \rightarrow 2$

intersect at a point.

Multiplying equation 1 by 3 and subtracting from equation 2 we get

$12\ x_1 + 6\ x_2 = 3000 \rightarrow 2$

$-12\ x_1 - 30\ x_2 = -6000 \rightarrow 1 X3$

$-24\ x_2 = -3000$ or $24\ x_2 = 3000$ or $x_2 = 3000/24 = 125$

Putting $x_2 = 125$ in equation 1 we get $4\ x_1 + 10 X 125 = 2000$

or $x_1 = (2000-1250)/4 = 750/4 = 187.5$

Therefore the point of intersection is (187.5,125)

Graphical Solution of Two-Variable L.P. Problem - An Indefinite Number of Optimal Solutions

The vertices of the feasible solution zone are(0,0),(250,0),(187.5,125) and (0,200)

At (0,0) Z=40X0+100X0=0

At (250,0) Z=40X250+100X0=10000

At (187.5,125) Z=40X187.5+100X125=20000

At (0,200) Z=40X0+100X200=20000

Since the optimal solution 20000 lies at 2 vertices, the same value lies at all points along the edge joining these points. Thus there are multiple optimal solutions for this problem

Graphical Solution of Two-Variable L.P. Problem – No Solution

Example: Maximize $Z=3x+2y$

Subject to $-2x+3y\leq9$

$3x-2y \leq-20$

Slope calculation $3x+2y=0$ or $3x=-2y$ or $x/y=-2/3$

Putting $x=0$ in constraint 1 and converting to equality we get $3y=9$ or $y=3$

Putting $y=0$ we get $-2x=9$ or $x=-4.5$

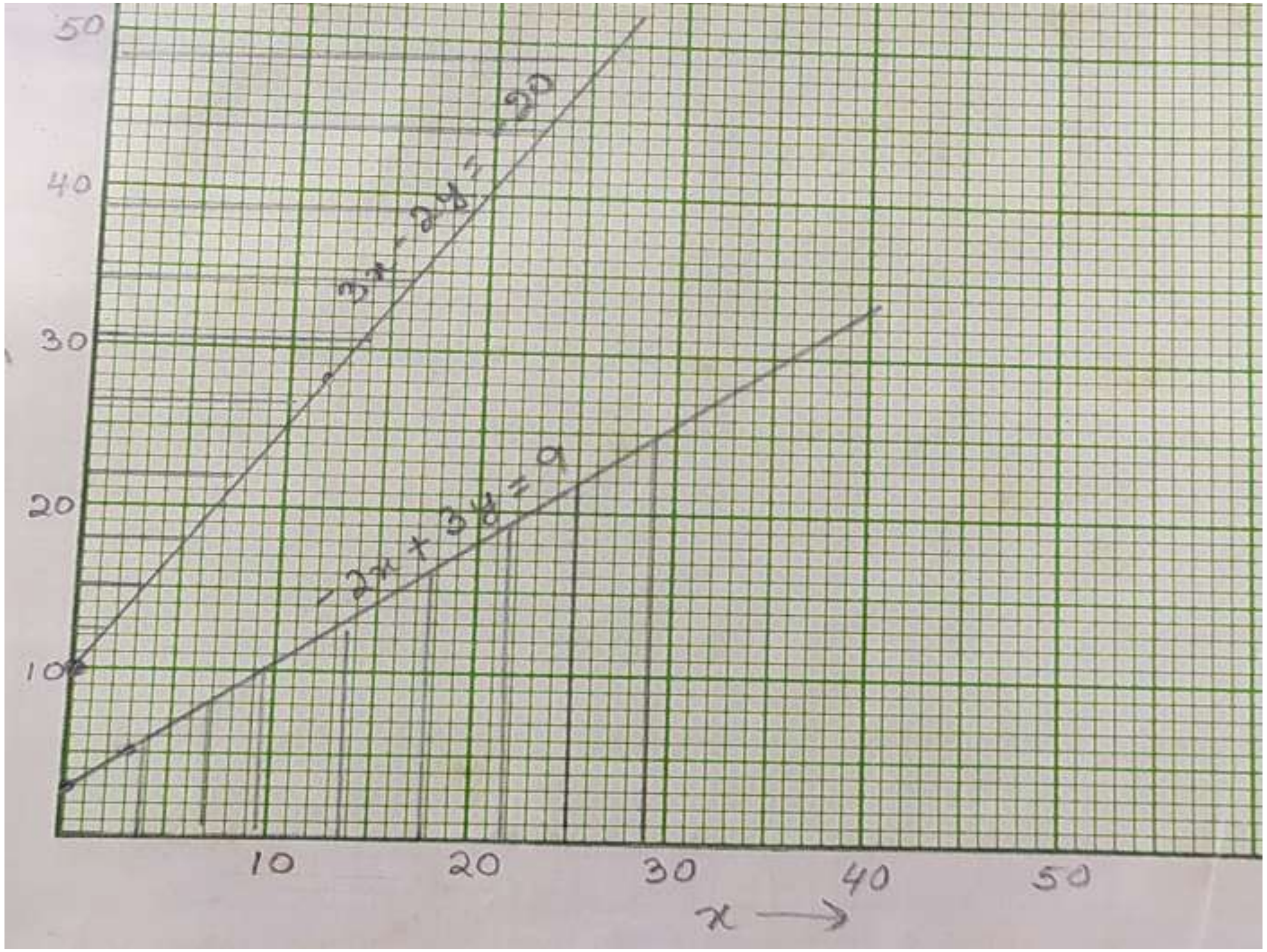
Therefore points (0,3) and (-4.5,0) lie on this line

Putting $x=0$ in constraint 2 and converting to an equality we get $-2y=-20$ or $y=10$

Putting $y=0$ we get $3x=-20$ or $x=-20/3=-6.67$

Therefore points (0,10) and (-6.67,0) lie on this line

Graphical Solution of Two-Variable L.P. Problem – No Solution



The feasible zone of the two constraints do not overlap. Hence there exists no solution to this problem.

Graphical Solution of Two-Variable L.P. Problem – Unbounded Solution

Example : Maximize $Z=5x_1+4x_2$

Subject to $x_1-2x_2 \le 1$

$x_1+2x_2 \ge 3$

$x_1 \ge 0,\ x_2 \ge 0$

Slope Calculation **5** $x_1+4x_2=0$ or $x_1=-4x_2/5$ or $x_1/x_2=-4/5$

Putting $x_1=0$ in constraint 1 and equating to 0, we get $-2x_2=1$ or $x_2=-1/2$

Putting $x_2=0$ in constraint 1 and equating to 0, we get $x_1=1$

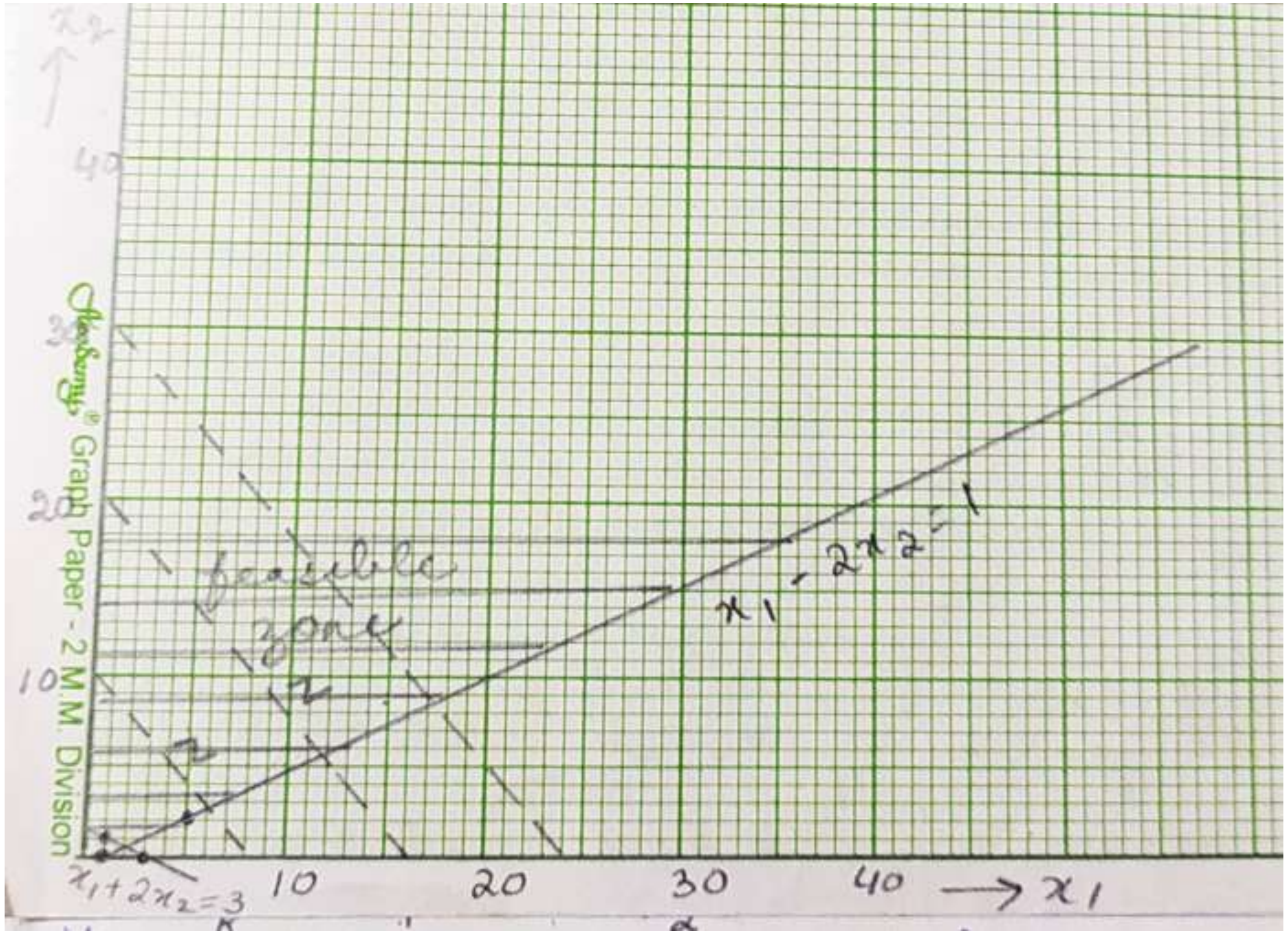
Two points on the line are (0,-1/2) and (1,0)

Putting $x_1=0$ in constraint 2 and equating to 0, we get $2x_2=3$ or $x_2=3/2=1.5$

Putting $x_2=0$ in constraint 2 and equating to 0, we get $x_1=3$

Two points on the line are (0,1.5) and (3,0)

Graphical Solution of Two-Variable L.P. Problem – Unbounded Solution



The solution is unbounded because Z increases towards the right and the feasible zone moves to infinity towards the right.

GRAPHICAL SOLUTION OF TWO-VARIABLE L.P. PROBLEM – A SINGLE SOLUTION

Example : Maximize $Z=5\ x_1 + 8x_2$

Subject to: $3\ x_1+5\ x_2= 18$

$5\ x_1 +3\ x_2=14$

$x_1 \geq 0,\ x_2 \geq 0$

Putting $x_1 =0$ in constraint 1 , we get $5\ x_2=18$ or $x_2 = 18/5=3.6$

Putting $x_2 =0$ in constraint 1, we get $3x_1 =18$ or $x_1 =18/3=6$

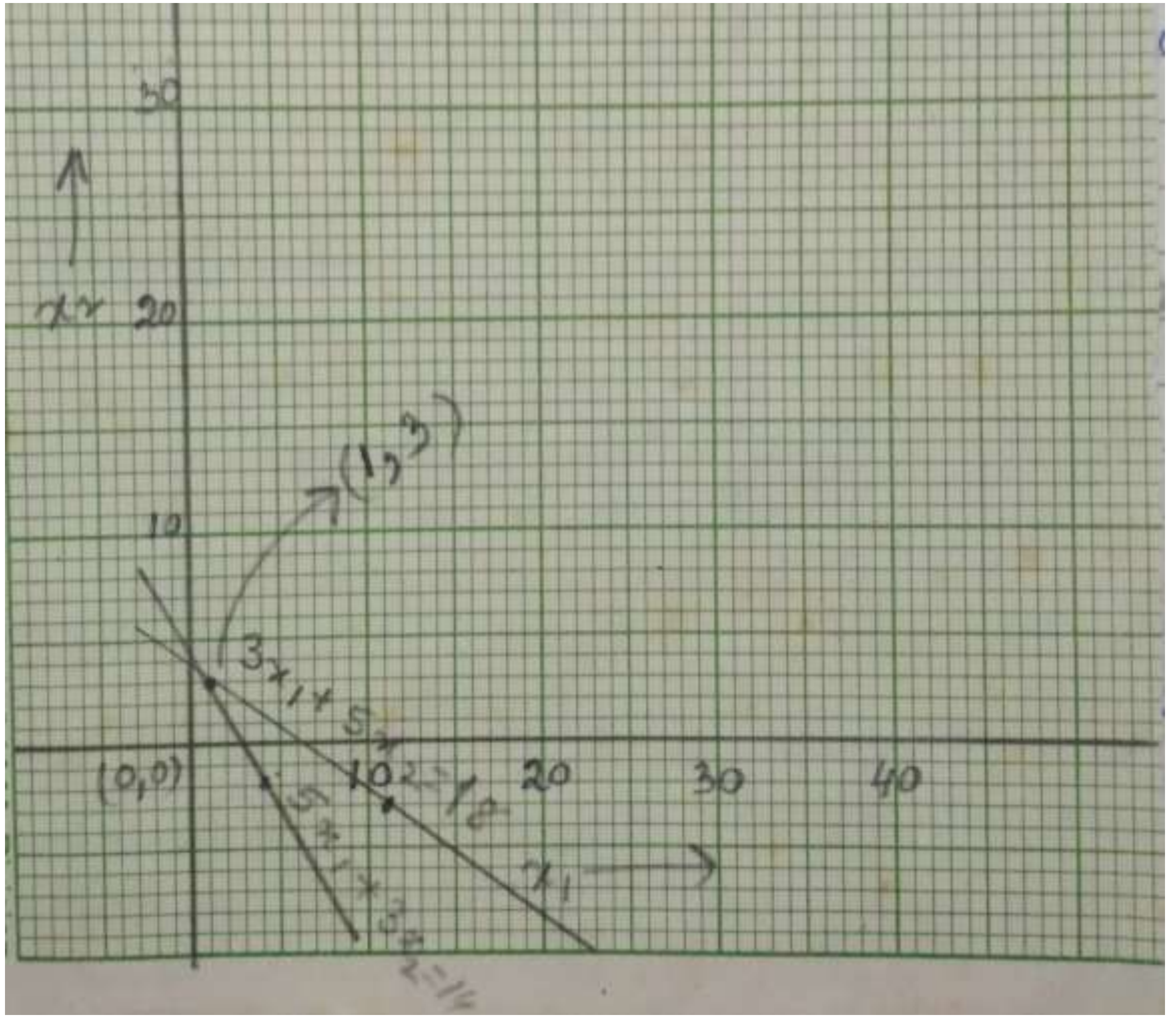
Two points on the line are (0,3.6) and (6,0)

Putting $x_1 =0$ in constraint 2, we get $3\ x_2=14$ or $x_2 = 14/3=4.67$

Putting $x_2 =0$ in constraint 2 , we get $5\ x_1 =14$ or $x_1 =14/5=2.8$

Two points on the line are (0,4.67) and (2.8,0)

Graphical Solution of Two-Variable L.P. Problem – Single Solution



The two equality constraints intersect at (1,3). Hence a single solution exists and no optimization is possible. The solution is 5X1+8X3=29

- TILL WE MEET AGAIN IN THE NEXT CLASS……….