# MB106
# QUANTITATIVE TECHNIQUES


**MODULE I**

LECTURE 3

Linear Programming: Graphical solution

# Graphical Solution of Two-Variable L.P. Problem

- A linear programming problem with two variables can be solved using the graphical method.
- A linear programming problem may have
  1. A definite and unique optimal solution
  2. An indefinite number of optimal solutions
  3. An unbounded solution
  4. No solution
  5. A single solution

# Graphical Method-Minimization Problem

- Example: A company has two bottling plants $P_1$ and $P_2$. Each plant produces three types of soft drinks A, B and C respectively. The number of bottles produced per day are as follows:

| | $P_1$ | $P_2$ |
|---|---|---|
| A | 15000 | 15000 |
| B | 3000 | 1000 |
| C | 2000 | 5000 |

A market survey indicates that during the next month there will be a demand of 20,000 bottles of A, 40000 bottles of B and 44,000 bottles of C. The operating costs per day per plants $P_1$ and $P_2$ are Rs. 600 and Rs.400. For how many days should each plant be run next month to minimize the production cost, while still meeting the market demand.

# Graphical Method-Minimization Problem

Let $P_1$ run for $x_1$ days

Let $P_2$ run for $x_2$ days

The objective-Minimization of production costs

Therefore the objective function is

Minimize $Z=600\, x_1 + 400\, x_2$

Subject to constraints

$15000x_1 + 15000\, x_2 \geq 20000$

$3000\, x_1 + 1000\, x_2 \geq 40000$

$2000\, x_1 + 5000\, x_2 \geq 44000$

$x_1 \geq 0, x_2 \geq 0$ → non-negativity constraints

# Graphical Method-Minimization Problem

Minimize $Z=600 x_1 +400 x_2$

Slope of the objective function :

$600 x_1 +400 x_2 =0$ or $600 x_1 = - 400 x_2$ or $x_1 / x_2 =-2/3$

$15000x_1 + 15000 x_2 = 20000$

Putting $x_1 =0$, $15000 x_2 = 20000$ or $x_2 =1.33$

Putting $x_2 =0$, $15000 x_1 = 20000$ or $x_1 =1.33$

Therefore (1.33,0) and (0,1.33) are two points lying on this line

$3000 x_1 + 1000 x_2 = 40000$

Putting $x_1 =0$, $1000 x_2 = 40000$ or $x_2 =40$

Putting $x_2 =0$, $3000 x_1 = 40000$ or $x_1 =13.33$

Therefore (13.33,0) and (0,40) are two points lying on this line

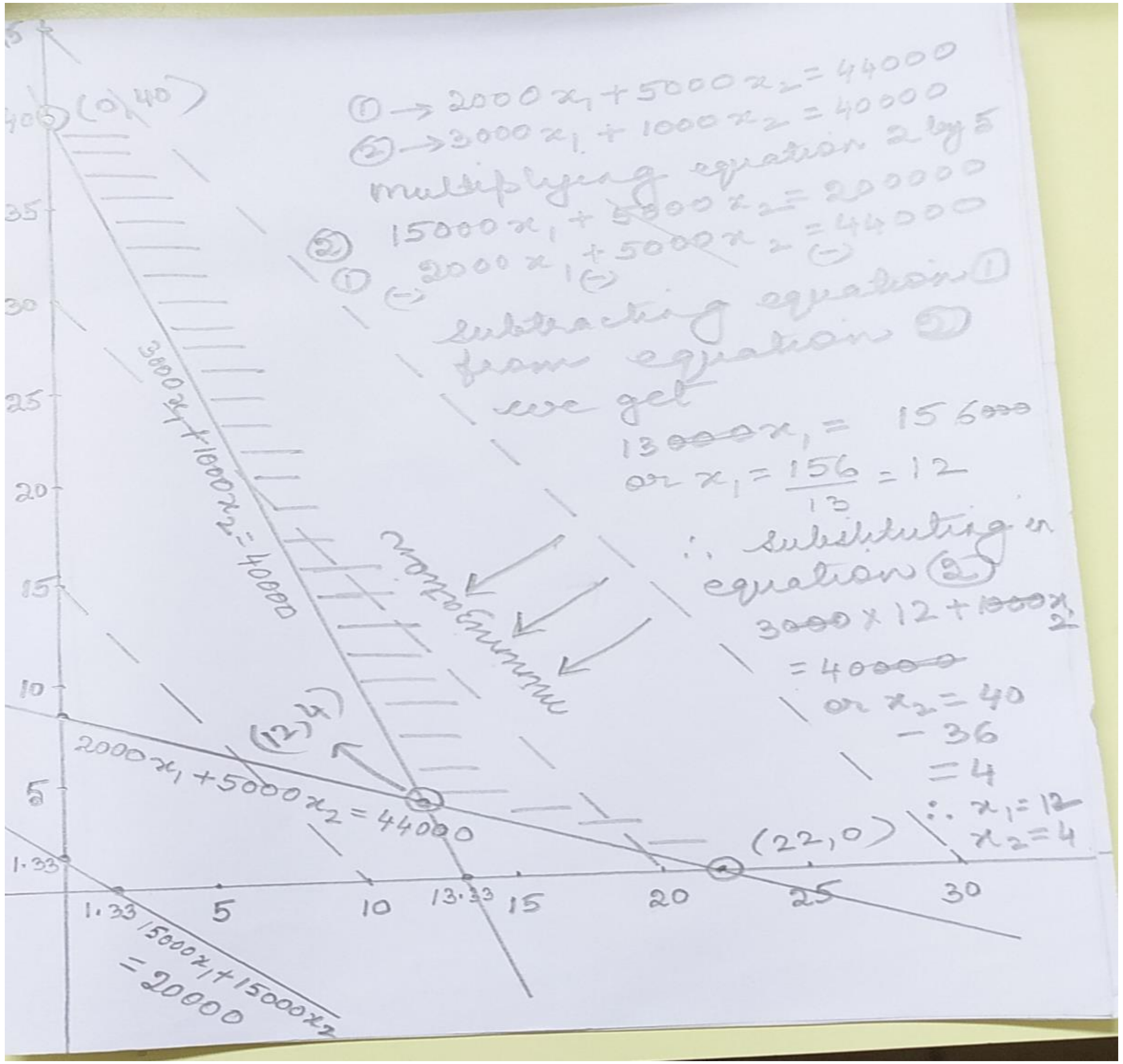
$2000 x_1 + 5000 x_2 = 44000$

Putting $x_1 =0$, $5000 x_2 = 44000$ or $x_2 =8.8$

Putting $x_2 =0$, $2000 x_1 = 44000$ or $x_1 =22$

Therefore (22,0) and (0,8.8) are two points lying on this line

# GRAPHICAL METHOD-MINIMIZATION PROBLEM




Considering the vertices of the solution zone

At (0,40)

Z=600X0+400X40=16000

At (30,0)

Z=600X30+400X0=18000

At (12,4)

Z=600X12+400X4=8800

Therefore minimum Z occurs at (12,4) and the value is 8,800

# GRAPHICAL METHOD-MAXIMIZATION PROBLEM

A firm manufactures two products A and B on which the profits earned per unit are Rs. 3 and Rs. 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and one minute on M2. Machine M1 is available for not more than 7 hrs 30 mins. Machine M2 is available for 10 hrs during any working day. Find the number of units of products A and B to be manufactured to get maximum profit.

# GRAPHICAL METHOD-MAXIMIZATION PROBLEM

- **To find: units of food types 1,2,3 and 4 to constitute the diet**
- Let $x_1$ be the number of units of A manufactured
- Let $x_2$ be the number of units of B manufactured
- Therefore $x_1 \geq 0$, $x_2 \geq 0$ →non-negativity constraints
- The objective-Maximization of cost subject to availability of machine time.
- Therefore the objective function is

Maximize $Z=3x_1+4x_2$

Slope of the objective function : $3x_1+4x_2=0$ or $x_1/x_2=-4/3$

# Graphical Method-Maximization Problem

- $x_1+x_2 \leq 450$ (7X60+30)
- $2x_1+x_2 \leq 600$ (10X60)

$x_1+x_2 = 450$

Putting $x_1=0$, $x_2=450$

Putting $x_2=0$, $x_1=450$

Therefore the two points on the line are (0,450) and (450,0)

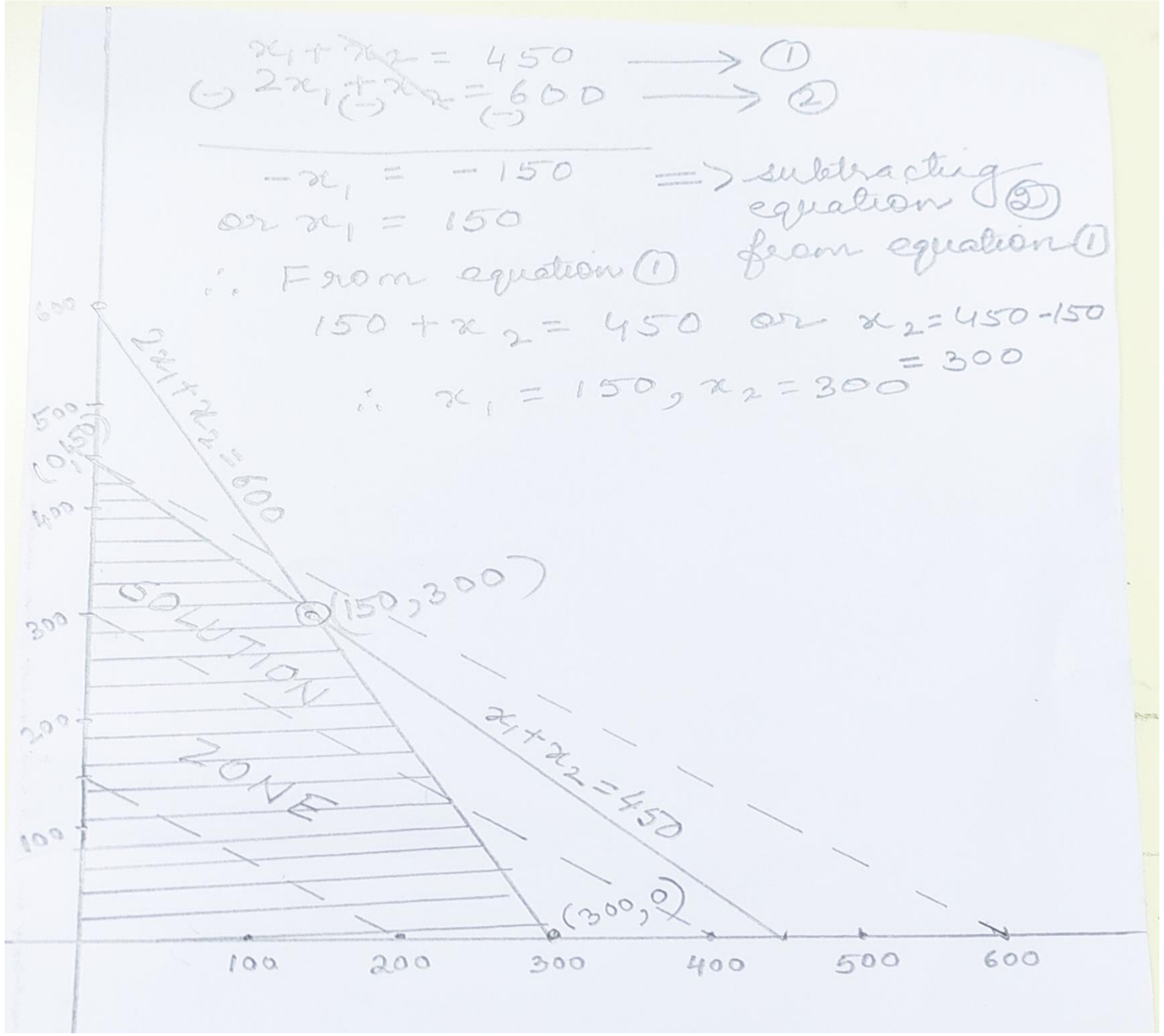
$2x_1+x_2 = 600$

Putting $x_1=0$, $x_2=600$

Putting $x_2=0$, $x_1=600/2=300$

Therefore the two points on the line are (0,600) and (300,0)

# GRAPHICAL METHOD-MAXIMIZATION PROBLEM




- Considering the vertices of the solution zone
- At (0,600) Z=3X0+4X450=1800
- At (300,0) Z=3X300+4X0=900
- At (150,300) Z=3X150+4X300=450+1200 =1650
- Therefore maximum Z occurs at (0,600) and the value is 1,800

• Till we meet again in the next class……….