# MB106
# QUANTITATIVE TECHNIQUES


**MODULE I**

LECTURE 2

Linear Programming: Formulating maximization/minimization problems

# FORMULATION OF LINEAR PROGRAMMING MODEL –A MAXIMIZATION PROBLEM

- A chemical company produces two products A and B. Each unit of product A requires 3 hrs on operation I and 4 hrs on operation II while each unit of product B requires 4 hrs on operation I and 5 hrs on operation II. Total available time for operations I and II are 20 and 26 hrs respectively. The production of each unit of product B results in two units of a by-product C at no extra cost. Product A sells at a profit of Rs. 10/- per unit while B sells at a profit of Rs. 20/- per unit. By-product C fetches Rs. 6/- if sold else its destruction cost is Rs. 4/- per unit. Forecast says not more than 5 units of C will sell. To maximise profits how many units of A and B should be produced?

# FORMULATION OF LINEAR PROGRAMMING MODEL –THE SOLUTION(OBJECTIVE FUNCTION)

- To find: units of A and B to be produced
- Let $x_1$ be the number of units of A produced
- Let $x_2$ be the number of units of B produced
- Let $x_z$ be the number of units of C produced= $x_3 + x_4$

  where $x_3$ is the number of unit of C sold and $x_4$ is the number of unit of C destroyed
- Therefore $x_1 \geq 0$, $x_2 \geq 0$ , $x_3 \geq 0$, $x_4 \geq 0$ →non-negativity constraints
- The objective-Maximization of profits earned
- Therefore the objective function is

$$\text{Maximize } Z=10\,x_1 +20\,x_2+6x_3 - 4\,x_4$$

# Formulation of Linear Programming Model –The Solution(Constraints)

- $3x_1 + 4x_2 \leq 20$ → time available on operation I
- $4x_1 + 5x_2 \leq 26$ → time available on operation 2
- $x_3 \leq 5$ → number of units of C sold
- If $x_2$ units of B are produced $2x_2$ units of C will be produced which is $x_3 + x_4$
- Therefore $2x_2 = x_3 + x_4$ → number of units of C produced or $-2x_2 + x_3 + x_4 = 0$

# Formulation of Linear Programming Model –The Model

Maximize $Z=10\ x_1 + 20\ x_2 + 6x_3 - 4\ x_4$

Subject to

$x_1 \geq 0,\ x_2 \geq 0\ ,\ x_2 \geq 0,\ x_4 \geq 0$ → non-negativity constraints

- $3x_1 + 4x_2 \leq 20$ → time available on operation I
- $4x_1 + 5x_2 \leq 26$ → time available on operation 2
- $x_3 \leq 5$ → number of units of C sold
- $-2x_2 + x_3 + x_4 = 0$ → number of units of C produced

# Formulation of Linear Programming Model –Problem 3→Minimization

- A dietician wants to decide the constituents of the diet for her patient that will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice has to be made from four different types of foods. The yields per unit of these foods are

| | yield/unit | | | cost(Rs) |
|---|---|---|---|---|
| Food type | proteins | fats | carbohydrates | per unit |
| 1 | 3 | 2 | 6 | 45 |
| 2 | 4 | 2 | 4 | 40 |
| 3 | 8 | 7 | 7 | 85 |
| 4 | 6 | 5 | 4 | 65 |
| **Minimum requirements** | 800 | 200 | 700 | |

# FORMULATION OF LINEAR PROGRAMMING MODEL –THE SOLUTION(OBJECTIVE FUNCTION)

- **To find: units of food types 1,2,3 and 4 to constitute the diet**
- Let $x_1$ be the number of units of food type 1
- Let $x_2$ be the number of units of food type 2
- Let $x_3$ be the number of units of food type 3
- Let $x_4$ be the number of units of food type 3
- Therefore $x_1 \geq 0$, $x_2 \geq 0$ , $x_3 \geq 0$ , $x_4 \geq 0$ →non-negativity constraints
- The objective-Minimization of cost subject to availability of required nutrients.
- Therefore the objective function is

$$\text{Minimize } Z=45\,x_1 + 40\,x_2 + 85x_3 + 65\,x_4$$

# Formulation of Linear Programming Model –The Solution(Constraints)

- $3x_1+4x_2+8x_3+6x_4 \geq 800$ → protein constraint
- $2x_1+2x_2+7x_3+5x_4 \geq 200$ → fat constraint
- $6x_1+4x_2+7x_3+4x_4 \geq 700$ → carbohydrate constraint

# FORMULATION OF LINEAR PROGRAMMING MODEL –THE MODEL

Minimize $Z=45\ x_1 + 40\ x_2 + 85x_3 + 65\ x_4$ Subject to

subject to

$x_1 \geq 0,\ x_2 \geq 0\ ,\ x_3 \geq 0,\ x_4 \geq 0$ →non-negativity constraints

$3x_1+4x_2+8x_3+6x_4 \geq 800$ →protein constraint

$2x_1+2x_2+7x_3+5x_4 \geq 200$ →fat constraint

$6x_1+4x_2+7x_3+4x_4 \geq 700$ →carbohydrate constraint

- Till we meet again in the next class……….